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Schooling Matters

OPPORTUNITY TO LEARN IN PISA 2012

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SCHOOLING MATTERS: OPPORTUNITY TO LEARN IN PISA 2012

OECD Education Working Paper no. 95

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ABSTRACT

Many international comparisons of education over the past 50 years have included some measure of students’ opportunity to learn (OTL) in their schooling. Results have typically confirmed the common sense notion that a student’s exposure in school to the assessed concepts, operationalized in some sort of time metric, is related to what the student has learned as measured by the assessment. What has not been demonstrated is a connection between the specifics of what students have encountered through schooling and their performance on any sort of applied knowledge assessment such as PISA. This paper explores this issue in 2012 PISA which, for the first time, included several OTL items on the student survey. OTL demonstrated a significant relationship with student performance on both the main paper-and-pencil literacy assessment as well as the optional computer-based assessment at all three levels – country, school and student. In every country at least one if not all three of the constructed OTL indices – exposure to word problems, formal mathematics topics, and applied mathematics problems – demonstrated a significant relationship to the overall PISA measure of mathematics literacy as well as the four sub areas of change and relationships, shapes and space, quantity, and uncertainty and data. Additionally, results indicated that variability in OTL was related to student performance having implications for equality of opportunity.

RÉSUMÉ

Ces 50 dernières années, nombre de comparaisons internationales de l'éducation ont inclus, sous une forme ou une autre, une mesure des possibilités d'apprentissage (*opportunity to learn*, OTL) des élèves au cours de leur scolarité. Les résultats ont généralement confirmé la notion de bon sens selon laquelle l'exposition des élèves dans la cadre scolaire aux concepts évalués, matérialisée sous forme de mesure temporelle, présente une corrélation avec les connaissances apprises par les élèves, telles que mesurées par l'évaluation. Ce qui n'a pas été démontré, en revanche, c'est le lien qui existe entre la nature des éléments spécifiques auxquels les élèves ont été exposés au cours de leur scolarité et leur performance à tout type d’évaluation des connaissances appliquées, telle que le PISA. Le présent document de travail étudie cette question dans le cadre de l’enquête PISA 2012 qui, pour la première fois, faisait figurer plusieurs items relatifs aux possibilités d’apprentissage dans son questionnaire destiné aux élèves. Il en ressort qu'il existe une corrélation significative entre les possibilités d'apprentissage et les résultats des élèves, tant dans l’évaluation papier-crayon principale que dans l’évaluation informatisée proposée à titre d’option, et ce à tous les niveaux – national, établissements d’enseignement et élèves. Dans tous les pays, au moins l'un des trois indices composites des possibilités d’apprentissage, si ce n'est tous (exposition aux problèmes lexicaux, exposition aux mathématiques formelles et exposition aux problèmes de mathématiques appliquées), présente une corrélation significative avec le niveau de compétence sur l’échelle PISA globale de culture mathématique, ainsi que sur les quatre sous-échelles *variations et relations*, *espace et formes*, *quantité* et *incertitude et données*. En outre, les résultats indiquent que la variation des indices des possibilités d’apprentissage influe sur la performance des élèves, laissant donc entrevoir des implications en termes d'égalité des chances.

SCHOOLING MATTERS: OPPORTUNITY TO LEARN IN PISA 2012

Over the past 50 years, international comparative studies of education have provided an international frame or lens through which participating countries can examine their own practices and policies. Although differences in student performance typically dominate the headlines the true value of such comparative efforts lie in the thoughtful consideration of policies related to the structure, organization, delivery, and content of instruction as a range of alternatives are brought to light.

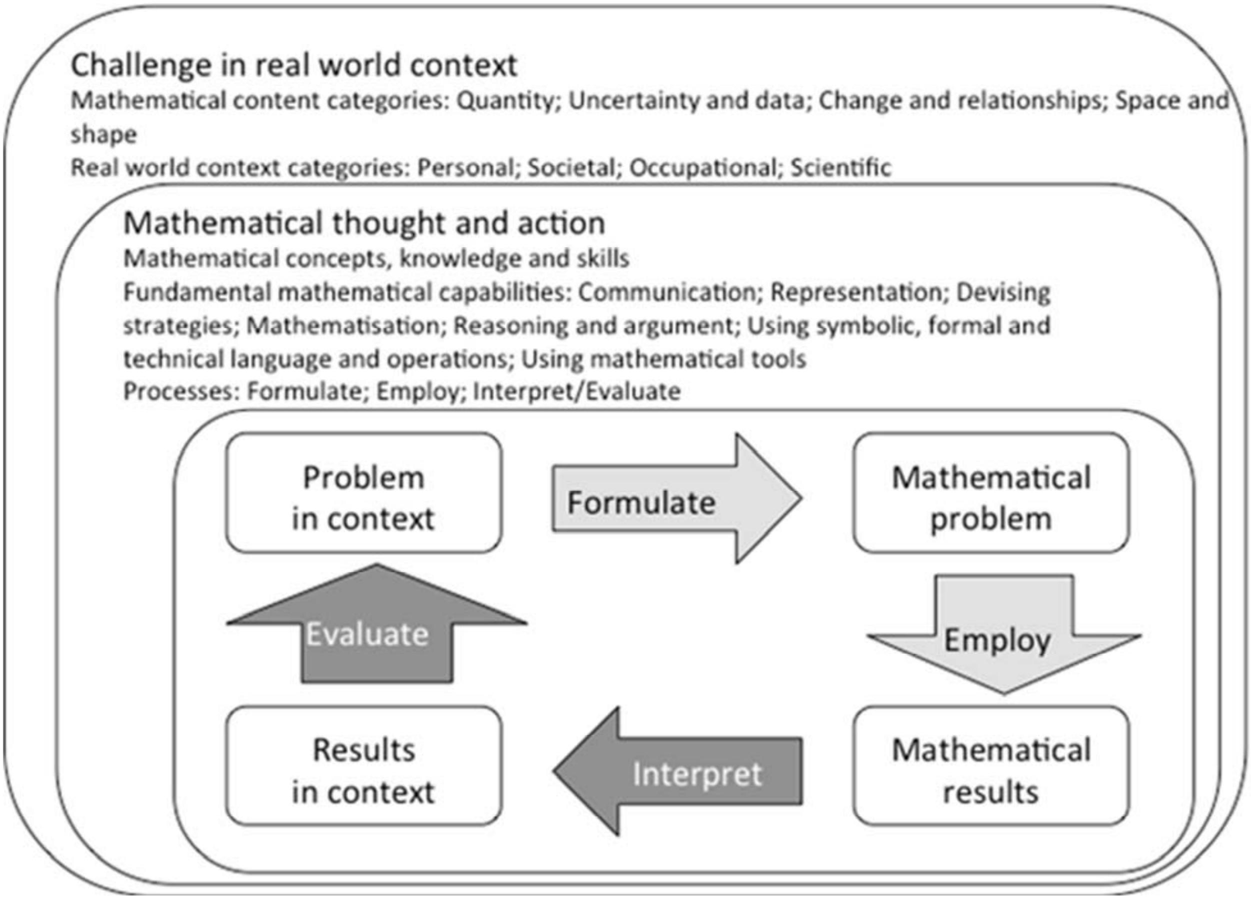
Two such major studies of mathematics are the Third International Mathematics and Science Study (TIMSS), sponsored by the International Association for the Evaluation of Educational Achievement (IEA), and the Program for International Student Assessment (PISA), sponsored by The Organisation for Economic Co-operation and Development (OECD) which has provided policy relevant data not only to its member countries but to many other countries and governmental/economic entities that have chosen to participate in the PISA assessments.

Both of these studies have distinctive frameworks that define the type and scope of the mathematics performance assessed. The TIMSS framework focused on the mathematics typically covered in the K-12 curriculum of the participating countries. The original 1995 TIMSS study spent over a year identifying the topics that were defined by the union of all those covered in over 40 countries in each of the grades 1-12 (Schmidt *et al*., 1997). Specific topics from the resulting framework were then selected to form the grade-specific assessment blueprints, *e.g*., grades four, eight, and twelve (or end-of-secondary).

In contrast, the PISA mathematics framework defines the mathematics to be assessed as the mathematics required to solve real world problems that young adults may encounter either in their current environment or in the future as citizens of their countries (see Figure 1). Such problems require the application of mathematics both in selecting the appropriate mathematical models and algorithms to solve the problem as well as the appropriate interpretation of the results of these mathematical algorithms. This focus is made clear in the PISA definition of literacy:

> *Mathematical literacy is an individual's capacity to formulate, employ, and interpret mathematics in a variety of contexts. It includes reasoning mathematically and using mathematical concepts, procedures, facts and tools to describe, explain and predict phenomena. It assists individuals to recognize the role that mathematics plays in the world and to make the well-founded judgments and decisions needed by constructive, engaged and reflective citizens (OECD, 2013b).*

Figure 1. A model of mathematical literacy in practice



Source: OECD, 2013b

The implication from the PISA framework is that 15 year olds need to be able to use their knowledge of mathematics first to recognize the mathematical nature of a problem and then to formulate the problem in mathematics terms. This transformation of the problem with all its inherent components as defined in the applied setting into a mathematics problem to be solved is crucial to the PISA definition of literacy. Having transformed and formulated the problem as a mathematics item the student must then solve it using the algorithms and procedures that are typically covered in the mathematics classes taught in schools. Finally, the student must take the results of the mathematics computation and place it back in its original applied setting and provide an appropriate interpretation that addresses the original problem. The PISA framework recognizes that the applied content of the problem can also be within mathematics itself by addressing a problem framed in a way such that the mathematics needed to solve it is not readily apparent. This rather complex process is what PISA measures and defines as mathematics literacy. Recognizing that the steps in the literacy model encompasses knowledge from various areas of mathematics, PISA identifies four content areas that are measured and reported as sub-scales: space and shape; quantity; change and relationships; and uncertainty and data. PISA also includes three subscales reporting on the processes that students do in solving the applied problem: formulating situations mathematically; employing mathematics concepts, facts procedures and reasoning; and interpreting, applying and evaluating mathematical outcomes.

Given this emphasis on measuring the application of mathematics, a reasonable question arises: is the formal mathematics content knowledge learned in school necessary to successfully complete these real world tasks or is there some other component that plays a role in solving such problems? *This paper addresses this question by exploring the role of schooling, i.e., students' learning opportunities in schools (OTL), in students' mathematical literacy as measured in PISA.*

Analyses of the 1995 TIMSS revealed that Opportunity to learn (OTL) measures, indicators of the schooling students have experienced, are related to student performance (Schmidt *et al*, 2001). OTL as a measure of schooling has a long history going back to the early 1960s. Essentially, OTL is based on the common sense notion that the time a student spends learning something is related to what the student learns. Bloom succinctly expressed this in his Thorndike award address stating, "All learning, whether done in school or elsewhere, requires time" (p. 682, Bloom 1974). Further, this idea about time is foundational to the concept of schools and schooling as the instructional/learning enterprise in schools is devoted to organizing and structuring students' time around learning specific subject matter content.

In the early 1960's, John B. Carroll was among the first to include time explicitly in his model of school learning (1963). In his model student learning was a function of both student factors (aptitude, ability, and perseverance) and factors essentially controlled by teachers (the time allocated for learning (OTL) and the quality of instruction). More recent work has defined OTL in terms of the specific content covered in classrooms and the amount of time spent covering these topics (Schmidt *et al*, 1999; Schmidt *et al*, 2001).

Using the concept of OTL as a measure of schooling, PISA 2012 included a small number of items on the student questionnaire in order to derive a set of OTL indices conceptually related to mathematics literacy (Cogan & Schmidt, 2013; OECD, 2013a). Three indices were developed from the student questionnaire items: formal mathematics, an indicator of the extent of student exposure to algebra and geometry topics in classroom instruction; word problems, an indicator of the frequency that students encountered word problems in their mathematics schooling; and applied mathematics, an indicator of the frequency students in their mathematics classes encountered problems that required the application of mathematics either in a mathematical situation or in an every day, real world context. Each of these indices were developed to have a range from 0 to 3. The meaning of these values derived from the underlying questionnaire items about the frequency students encountered specific topics/situations: 0, "never"; 1, "rarely; 2, "sometimes; and 3, "frequently" (OECD, 2013a). The specific topics listed in the student questionnaire incorporated into the formal mathematics OTL indicator addressed the type of mathematics typically covered in grades 8-10.

These three indices provide the opportunity for the first time to explore the relationship between schooling and the mathematical literacy measured by PISA. The indices and the analyses presented here are not exhaustive; it is possible to have collected other more comprehensive OTL information and, indeed, the PISA field trial did include many more items than could be included in the main survey questionnaires (Cogan & Schmidt, 2014). In addition, the method PISA uses to sample students, randomly sampling all 15-year-olds within sampled schools, doesn't easily support the collection of class or classroom specific OTL information from teachers. Consequently, all PISA OTL data is student-reported.

Some may question the reliability of such student-reported information but students at this age and stage of development can be expected to be fairly reliable informants on what they have experienced in their classroom instruction and learning. Indeed, from a phenomenological viewpoint, if a 15-year-old student doesn't recall having encountered something in their classroom instruction, it would be surprising to find that the student actually knew much about that topic. TIMSS (1995) collected extensive OTL data from teachers of 12-14 year olds. Using those country-level data on the 28 countries that also participated in PISA (2012), there was a correlation of .59 between the two OTL measures. Recognizing the large time lapse and the difference in age groups, this still tends to add credence to the student ratings.

Drawing upon previous analyses of OTL and student performance in international comparative studies of mathematics, the hypothesis underlying the analyses reported here is that variability in OTL within a country derives from differences in content exposure which, in turn, stems mainly from students taking different mathematics courses as well as from different teachers teaching the same course. Given that in PISA students are randomly drawn from the school sampling frame without regard to course or classroom, student is confounded with both teacher and course type. In this situation it is appropriate to understand variation in student OTL as the variation that 15-year-olds have experienced in their opportunities to learn mathematics in their school. However, the source of this student-level variation in OTL is confounded among several factors including: the specific opportunities students have experienced individually because they are in different courses; taking the same course but with different teachers; and the unique history of courses taken up to this point at previous grades.

Conceptually, the three OTL indices may be related to the PISA literacy test at each of three levels: country, school and student. Fitting the data simultaneously using a statistical model that incorporates all three levels has the advantage of statistically controlling the relationship at each level for the relationship among the variables in the model for the other level(s). A three-level model relating the three OTL indices to the PISA literacy measure revealed statistically significant relationships for all three OTL variables at all three levels: country, school and student. Conceptually this means that the observed variation in the OTL indices is related to (correlated with) the observed variation in the PISA mathematics literacy score across individual students, schools, and countries. The precise nature of this relationship for any one country or any one of the OTL indices, however, requires further exploration. The following sections explore the relationship between OTL and PISA mathematics literacy at the country, school, and individual student level.

Relating OTL to Literacy at the Country Level

Discussion in this section focuses on the 34 OECD countries augmented with eight additional countries, the eight largest Non-OECD countries/economies in the world based on their GDP that participated in PISA 2012. These eight are: the Russian Federation, Brazil, Indonesia, Argentina, Colombia, Malaysia, Thailand and Chinese Taipei.

Although discussion is focused on these 42 countries, referred to as the OECD$^+$ countries, some discussions mention other major economies such as Hong Kong-China and Singapore. Nonetheless, all the major analyses presented in the tables include all 64 countries/economic entities that participated in PISA 2012.

Figure 2 provides a visualization of the extent of the variation across the 64 countries in terms of the opportunities to learn they provided on average. Word problems and formal mathematics demonstrated substantial variation among the 64 means ranging over a considerable part (1.2 to 2.3) of the scale. Applied mathematics demonstrated somewhat less variation. Nevertheless, the average values across the 64 PISA 2012 participants for all three indices ranged between “rarely” to “sometimes” providing such opportunities.

Figure 2. Boxplot of Country Means for all OECD and Other Participating Countries in PISA 2012

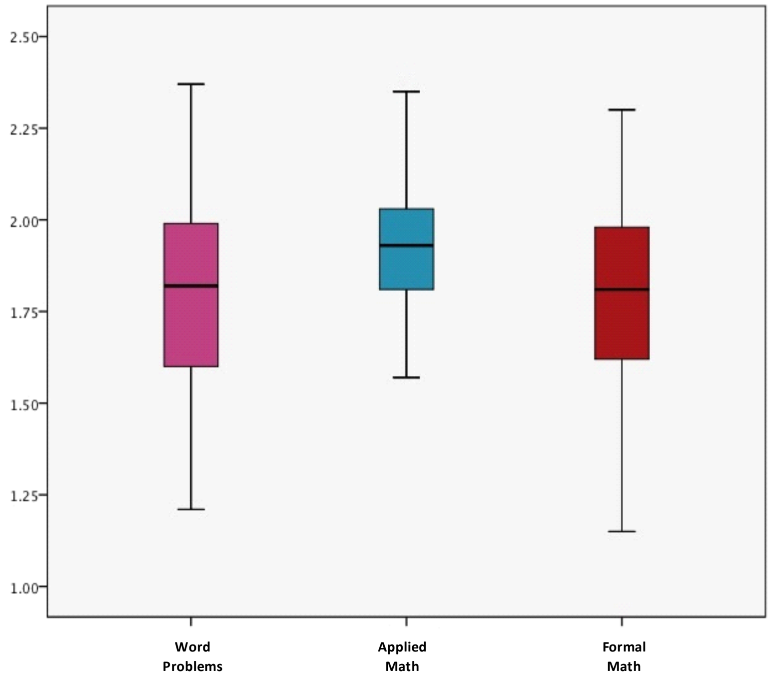


Table 1 lists each OECD country/ OECD partner together with their average for each of the OTL indicators. The mean across the OECD+ countries was 1.8 for word problems, 1.7 for formal mathematics and 1.9 for applied mathematics. Japan, the Republic of Korea and the Russian Federation were the three countries that were perceived by their 15 year olds on average as providing the largest amount of opportunity to learn formal mathematics among the OECD+ countries. Means for Shanghai-China, Singapore and Macao-China, indicate that they provided even a greater amount of formal mathematics opportunities. In contrast, Switzerland, Argentina, Brazil and Iceland were the four OECD+ countries that provided the least amount of formal mathematics OTL.

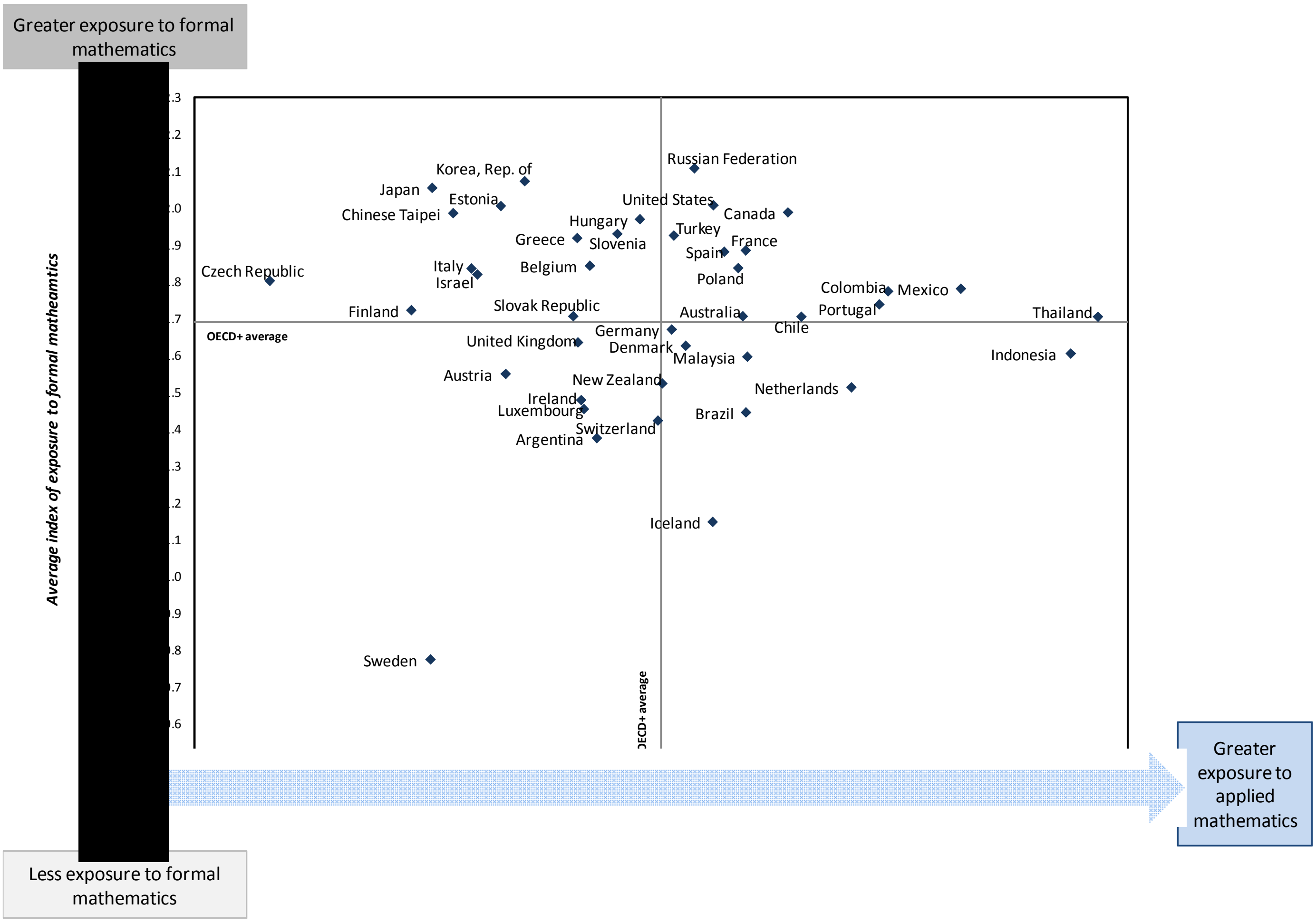
Results were substantially different for applied mathematics. Indonesia and Thailand provided the largest amount of time related to encountering applied problems in their schooling as noted by their 15 year olds. The Czech Republic provided the fewest school-related encounters with applied mathematics among OECD+ countries. Among the three economies that provided the most OTL related to formal mathematics, Macao-China and Shanghai-China were among those providing the least OTL related to applied mathematics. Singapore, on the other hand, was above the OECD+ average both for applied and formal mathematics.

Figure 3 divides the OECD+ countries into four groups related to the amount of opportunities to learn in both formal and applied mathematics. The OECD+ averages are used to create the four quadrants. As one might expect, this graph suggests that there may be a trade-off between the two types of opportunities. Although there were countries in the group defined by being above the OECD+ average on both indices, there were no countries that are in the top five both in the amount of formal and applied mathematics except for Canada.

Table 1. Country Level Means for Performance and the Three OTL Indices

Country	**Word Problems**	**Formal Math**	**Applied Math**	**Math Literacy Score**
	Mean (se)	*Mean (se)*	*Mean (se)*	*Mean (se)*
Australia	1.8 (0.0)	1.7 (0.0)	2.0 (0.0)	504 (2)
Austria	2.1 (0.0)	1.5 (0.0)	1.8 (0.0)	506 (3)
Belgium	1.9 (0.0)	1.8 (0.0)	1.9 (0.0)	515 (2)
Canada	2.0 (0.0)	2.0 (0.0)	2.1 (0.0)	518 (2)
Chile	2.0 (0.0)	1.7 (0.0)	2.1 (0.0)	422 (3)
Czech Republic	1.6 (0.0)	1.8 (0.0)	1.6 (0.0)	499 (3)
Denmark	1.9 (0.0)	1.6 (0.0)	2.0 (0.0)	500 (2)
Estonia	1.8 (0.0)	2.0 (0.0)	1.8 (0.0)	520 (2)
Finland	2.1 (0.0)	1.7 (0.0)	1.7 (0.0)	519 (2)
France	2.1 (0.0)	1.9 (0.0)	2.0 (0.0)	496 (2)
Germany	2.0 (0.0)	1.7 (0.0)	2.0 (0.0)	514 (3)
Greece	1.3 (0.0)	1.9 (0.0)	1.9 (0.0)	453 (2)
Hungary	2.0 (0.0)	2.0 (0.0)	1.9 (0.0)	478 (3)
Iceland	2.4 (0.0)	1.1 (0.0)	2.0 (0.0)	493 (2)
Ireland	1.8 (0.0)	1.5 (0.0)	1.9 (0.0)	501 (2)
Israel	1.7 (0.0)	1.8 (0.0)	1.8 (0.0)	467 (5)
Italy	1.7 (0.0)	1.8 (0.0)	1.8 (0.0)	485 (2)
Japan	1.6 (0.0)	2.1 (0.0)	1.7 (0.0)	537 (4)
Korea, Rep. of	1.7 (0.0)	2.1 (0.0)	1.8 (0.0)	554 (5)
Luxembourg	2.0 (0.0)	1.5 (0.0)	1.9 (0.0)	490 (1)
Mexico	1.8 (0.0)	1.8 (0.0)	2.2 (0.0)	413 (1)
Netherlands	1.6 (0.0)	1.5 (0.0)	2.1 (0.0)	522 (3)
New Zealand	1.6 (0.0)	1.5 (0.0)	2.0 (0.0)	500 (2)
Norway	1.8 (0.0)		1.8 (0.0)	490 (3)
Poland	2.0 (0.0)	1.8 (0.0)	2.0 (0.0)	518 (4)
Portugal	1.5 (0.0)	1.7 (0.0)	2.2 (0.0)	486 (4)
Slovak Republic	2.0 (0.0)	1.7 (0.0)	1.9 (0.0)	482 (3)
Slovenia	2.1 (0.0)	1.9 (0.0)	1.9 (0.0)	501 (1)
Spain	2.2 (0.0)	1.9 (0.0)	2.0 (0.0)	485 (2)
Sweden	1.9 (0.0)	0.8 (0.0)	1.7 (0.0)	478 (2)
Switzerland	2.1 (0.0)	1.4 (0.0)	1.9 (0.0)	531 (3)
Turkey	1.3 (0.0)	1.9 (0.0)	2.0 (0.0)	447 (5)
United Kingdom	1.9 (0.0)	1.6 (0.0)	1.9 (0.0)	494 (3)
United States	1.8 (0.0)	2.0 (0.0)	2.0 (0.0)	481 (4)
OECD Average	***1.9 (0.0)***	***1.7 (0.0)***	***1.9 (0.0)***	***494 (0)***
Argentina	1.6 (0.0)	1.4 (0.0)	1.9 (0.0)	388 (3)
Brazil	1.5 (0.0)	1.4 (0.0)	2.0 (0.0)	391 (2)
Chines Taipei	1.5 (0.0)	2.0 (0.0)	1.7 (0.0)	559 (3)
Colombia	1.9 (0.0)	1.8 (0.0)	2.2 (0.0)	377 (3)
Indonesia	1.9 (0.0)	1.6 (0.0)	2.3 (0.0)	376 (4)
Malaysia	1.8 (0.0)	1.6 (0.0)	2.0 (0.0)	421 (3)
Russian Federation	2.0 (0.0)	2.1 (0.0)	2.0 (0.0)	482 (3)
Thailand	1.9 (0.0)	1.7 (0.0)	2.4 (0.0)	427 (3)
OECD+ Average	**1.8 (0.0)**	**1.7 (0.0)**	**1.9 (0.0)**	**482 (0)**
Albania	1.9 (0.0)	2.1 (0.0)	2.2 (0.0)	395 (2)
Bulgaria	1.5 (0.0)	2.0 (0.0)	1.9 (0.0)	438 (4)
Costa Rica	1.6 (0.0)	1.5 (0.0)	1.7 (0.0)	407 (3)
Croatia	2.0 (0.0)	2.1 (0.0)	1.8 (0.0)	471 (3)
Hong Kong SAR	1.4 (0.0)	1.8 (0.0)	1.8 (0.0)	561 (3)
Jordan	2.2 (0.0)	2.2 (0.0)	2.2 (0.0)	386 (3)
Kazakhstan	1.9 (0.0)	2.0 (0.0)	2.2 (0.0)	432 (3)
Latvia	1.7 (0.0)	2.0 (0.0)	1.9 (0.0)	490 (3)
Liechtenstein	2.2 (0.1)	1.5 (0.0)	2.0 (0.0)	534 (4)
Lithuania	1.6 (0.0)	1.6 (0.0)	1.9 (0.0)	478 (3)
Macao-China	1.2 (0.0)	2.2 (0.0)	1.6 (0.0)	538 (1)
Montenegro	2.0 (0.0)	1.9 (0.0)	1.9 (0.0)	410 (1)
Peru	1.9 (0.0)	1.8 (0.0)	2.1 (0.0)	368 (4)
Qatar	1.7 (0.0)	1.7 (0.0)	2.0 (0.0)	377 (1)
Romania	1.9 (0.0)	2.0 (0.0)	2.1 (0.0)	445 (4)
Serbia	1.5 (0.0)	2.1 (0.0)	1.8 (0.0)	449 (3)
Shanghai-China	1.3 (0.0)	2.3 (0.0)	1.6 (0.0)	612 (3)
Singapore	1.6 (0.0)	2.2 (0.0)	2.0 (0.0)	573 (1)
Tunisia	1.6 (0.0)	1.2 (0.0)	2.1 (0.0)	388 (4)
United Arab Emirates	1.8 (0.0)	2.1 (0.0)	2.1 (0.0)	434 (2)
Uraguay	1.3 (0.0)	1.7 (0.0)	1.7 (0.0)	409 (3)
Vietnam	1.2 (0.0)	2.0 (0.0)	1.7 (0.0)	511 (5)

Figure 3. Average Index of Exposure to Formal Mathematics for OECD+ Countries by Average Index of Applied Mathematics Exposure



A further examination of the means of the three OTL variables across countries suggests several anomalies. Such anomalies can be identified by using a median polish, which takes into account the general level of OTL found in a country and the typical amount of OTL for each of the three indices across countries. Chinese Taipei, for example, appears to trade-off between traditional word problems typically found in mathematics textbooks and applied problems. They had a higher exposure to applied mathematics and a lower exposure to word problems than would be expected given the overall amount of OTL provided and the pattern of OTL for these two variables across countries. Japan, the Republic of Korea, the United States, Turkey, and Greece had a similar pattern but by contrast Switzerland, Austria, Sweden, Iceland, and Luxembourg had the exact opposite relying more heavily on word problems with less emphasis on applied mathematics than would be expected.

The median polish results point out anomalies to the otherwise general pattern of applied mathematics and word problems being positively related which is supported by their statistically significant correlation of .39. Applied mathematics, on the other hand, is essentially uncorrelated with formal mathematics (-.03).

We now turn to the relationship of schooling as measured by two of the OTL variables to student performance on the literacy assessment. Table 1 also provides the mean student performance for each country which was defined using the first plausible value. The OECD+ and the OECD means were essentially identical for the three OTL variables (word problems varied only by .1). However, the OECD mathematics literacy mean was higher than that of the OECD+ set of countries – 494 vs. 482.

The results from a simple regression model exploring the relationship of formal and applied mathematics to student performance for the 41 OECD+ countries (Norway had to be excluded due to missing data on the formal mathematics index) are summarized in Table 2. The relationship for formal mathematics to literacy was not statistically significant (p < .2) although the relationship was positive. In contrast applied mathematics was statistically significantly related to performance. The relationship was quadratic in nature.

Table 2. Summary of Country Level Regressions: Estimated Regression Coefficients for Different Literacy Measures

	Number of Countries	***Applied Math***	***Applied. Math Squared***	***Formal Math***	*R Square*
Math Literacy	41	1082 **	-324 **	38 *	0.55
Change	*41*	*1200 ** *	*-360 ** *	*51 * *	*0.56*
Quantity	41	1072 **	-324 **	30	0.56
Shapes & Space	*41*	*1096 ** *	*-328 ** *	*50 * *	*0.51*
Data	41	1081 **	-318 **	27	0.52
Employ	*41*	*1089 ** *	*-327 ** *	*41 * *	*0.56*
Formulate	41	1083 **	-328 **	40	0.52
Interpret	*41*	*1129 ** *	*-334 ** *	*31*	*0.55*
Computer Problem Solving	43	804 **	-251 **	38	0.29
Computer Math	*31*	*588 ** *	*-208 ** *	*60 ** *	*0.59*

** p<.05 ** p<.01*

Figure 4 graphically displays the relationship of applied mathematics to performance on the literacy test at the country level. It is somewhere between 1 and 2 ("rarely" and "sometimes" as response categories in the questionnaire) that country performance began to decrease.

Putting both OTL indices in the same analysis (see Table 2) resulted in a significant relationship for both formal and applied mathematics. The estimate of the strength of the predicted change in performance for a one unit change in the formal mathematics index was about a third of a standard deviation. When considered together the mean frequency of encounters with applied mathematics – in terms of problems in students' lessons and on the assessments they have taken – was related to average performance on the literacy assessment. This relationship was quadratic as illustrated in Figure 3.

Figure 4. Average PISA Mathematics Literacy Performance for OECD+ Countries by Average Applied Mathematics Exposure

The relationship of formal mathematics to performance was more straight-forward suggesting a simple linear relationship with performance when controlling for the relationship of applied mathematics implying that the more familiar and knowledgeable the average student in a country is with the key concepts of algebra and geometry, the better the mean performance of the country on the PISA literacy assessment.

Four mathematics content areas were described in the PISA framework resulting in the reporting of four sub-scores: space and shape; quantity; change and relationship; and uncertainty and data. The nature of the relationship of formal and applied mathematics OTL to student performance was essentially the same for two of these subtest areas (see Table 2). The relationship for space and shape and change and relationship essentially mirrored that for the overall literacy performance.

However, for quantity, and uncertainty and data, the quadratic relationship with applied mathematics was present but formal mathematics was only marginally significant ($p < .08$ and $p < .11$, respectively). Since the formal mathematics OTL measure is based on the average frequency of encountering topics related to algebra and geometry this result is not surprising.

Table 2 also presents the results for the three process subscales. The extent of the exposure students had with respect to formal mathematics in their schooling was related only to the employ aspect of the literacy model although it was marginally significantly related to the other two process scales ($p < .07$ and $p < .11$). It is at the employ stage, after the original problem has been phrased mathematically that the use of formal mathematics comes into play, however it is surprising that formal mathematics is not related to the formulate process score. On the other hand, applied mathematics was related to all three stages of the model being the only OTL variable related to the formulation and the interpretation of the problem subscales.

Results for the PISA computer-based assessment of mathematics literacy (see Table 2) also mirrored that of the relationship of OTL to literacy suggesting a degree of consistency between the paper-and-pencil measurement of mathematics literacy and that measured in the computer-based assessment. This computer-based assessment was optional and was taken by students in 31 countries.

Taken by students in 43 countries as a special option, was a problem-solving competency assessment. It was designed to assess literacy (related to reading), numeracy and problem solving in a technology environment. The results of conducting an analysis relating the same two OTL variables to performance on this test produced some interesting results. Although the assessment relates to numeracy it was not designed to be a mathematics problem solving test. Consequently, it was interesting and thought-provoking to examine the results of this analysis (also summarized in Table 2). Formal mathematics was not statistically significantly related ($p < .14$) to performance on the problem solving assessment, but OTL related to the frequency with which the typical student in a country had encountered applied mathematics was statistically significantly related to performance across the participating 43 countries. The nature of the relationship was again, quadratic. The report on the problem solving assessment will not be published until March 2014 at which time this apparent relationship can be more thoroughly examined. Why there is a relationship of opportunity to learn applied mathematics problems to a general problem solving assessment awaits further analysis. Perhaps at this point it can be pointed out that these results are consistent with those who have suggested that mathematics is an area of schooling that helps to support the development of problem-solving competencies more generally.

Relating OTL to Literacy Within Each Country

The three-level hierarchical analysis mentioned earlier indicated significant relationships of the three OTL indices to the PISA mathematics literacy paper and pencil assessment. The relationship was significant at all three levels: across countries, across schools within countries, and across students within schools. The preceding section presented the estimated relationships these OTL indices had with the various PISA scores across participating countries. In this section the relationships are examined within each country using a two-level hierarchical model.

In this section two related issues are explored. First, is the characterization of the relationship of word problems, formal and applied mathematics to performance within each of the PISA countries. The second related issue, really a subset of the first one, is whether the within-country relationships are present at the level of the school, the individual student, or both. These are important issues as the policy implications are different for the relationship of OTL to performance at each of the two levels.

A between-school relationship implies that differences across schools within a country in opportunity have a significant relationship to performance. Such a pattern can be due to a policy of tracking which identifies different types of schools for 15 year olds with policies determining which students can attend which type of school. Tracking is usually based on student ability with the different types of schools leading to different post secondary options. Social class segregation, which is often a result of housing patterns is another root cause of school level differences. The differentiation, however arrived at, essentially results in different content exposure. On the other hand, differences within schools focuses the issue of inequality related to OTL at the student level some of which could be the result of within-school tracking into different courses or due to student choice as to what courses they have chosen to take or related to teacher differences in instruction or some combination of the three. Again, the discussion that follows centers primarily on the OECD+ countries.

Table 3 lists all 64 participating countries and indicates which of the three OTL variables were significantly related to performance on the PISA mathematics literacy score at the student level, the school level, or both. The table also indicates whether the relationship was linear or quadratic. For the vast majority of OECD+ countries, applied mathematics was significant at the individual student level, the school level, or both. In most cases, the nature of this significant relationship was quadratic. This implies that the quadratic relationship reported in the previous section at the country level was not idiosyncratic to country differences but was characteristic of the within-country relationship at the individual student level as well as the school level. Formal mathematics was significantly related to performance at both levels in every OECD+ country. In addition, word problems were statistically significant at the student, school, or both levels in nearly all OECD countries (27 out of 34) and in five out of the eight additional countries making up the OECD+.

Figures 5, 6, and 7 display the estimated values for the regression coefficients for the three OTL variables at the student level for all countries. For the OECD+ countries, the average estimated regression coefficient for formal mathematics at the student level was 49, suggesting an effect size of about one-half of a standard deviation. This implies that students having often encountered algebra and geometry topics in their schooling as compared to encountering them only a few times had, on average, a higher performance of 50 points. Both word problems and applied mathematics had a statistically significant relationship at the student level in about three-fourths of the OECD+ countries. However, the effect sizes were much smaller. Figure 7 lists the estimated coefficients for the quadratic term in the applied mathematics regression. The negative values create the shape portrayed in Figure 8.

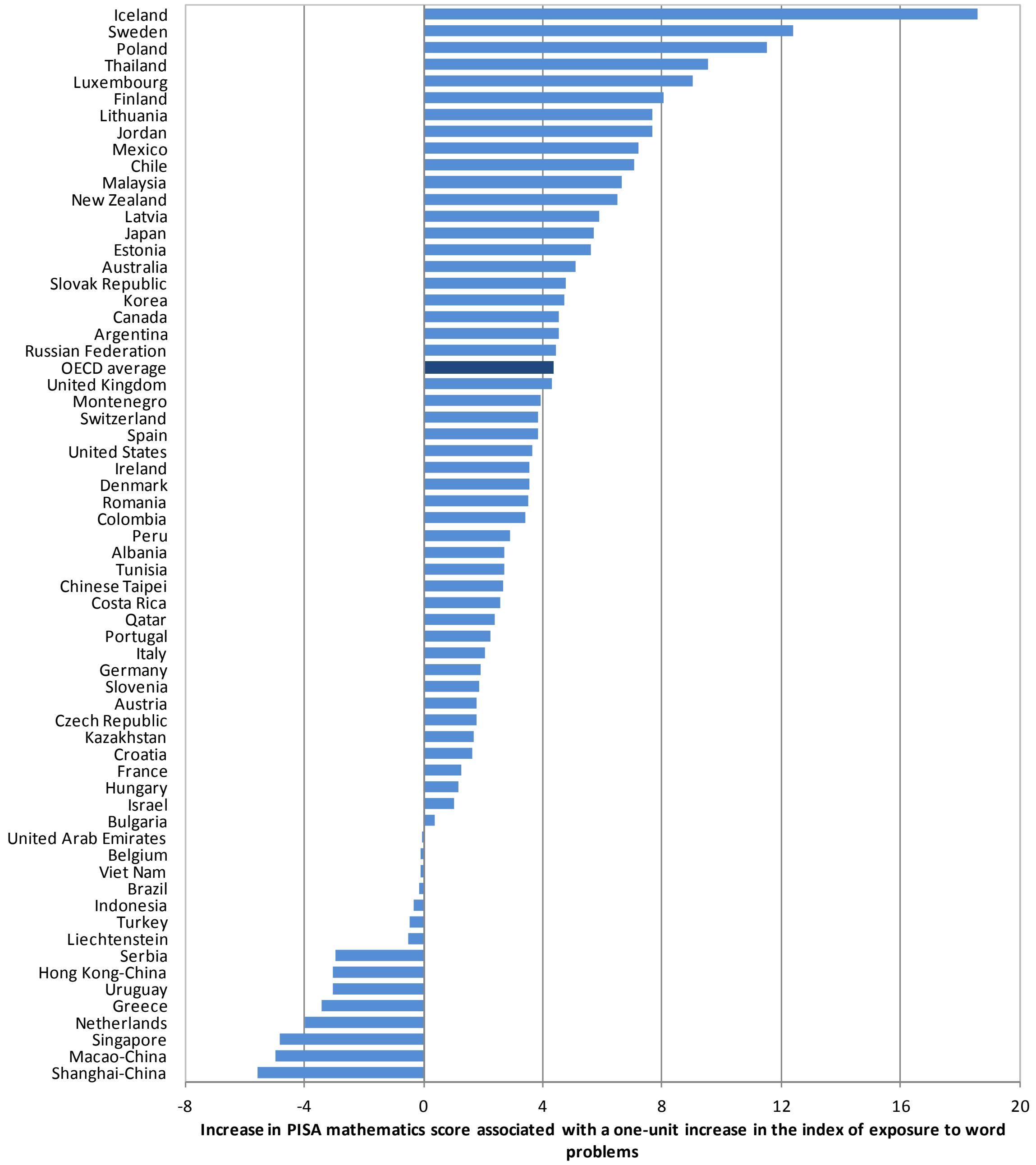
Table 3. Country-level Regressions Between Opportunity to Learn Variables and Mathematics Performance at the Student and School Levels

	Student			School		
	Word Problems	Applied Mathematics	Formal Mathematics	Word Problems	Applied Mathematics	Formal Mathematics
OECD						
Australia	L		L	L		L
Austria			L	L	Q	L
Belgium			L			L
Canada	L	Q	L	L	Q	L
Chile	L	Q	L	L	Q	L
Czech Republic			L	L		L
Denmark	L	Q	L		Q	L
Estonia	L		L	L	Q	L
Finland	L	Q	L	L	L	L
France		Q	L			L
Germany			L	L		L
Greece	L		L	L		L
Hungary			L			L
Iceland	L	Q	L	L	Q	L
Ireland	L	Q	L		L	L
Israel			L		Q	L
Italy	L	Q	L	L	Q	L
Japan	L	Q	L	L		L
Korea	L		L			L
Luxembourg	L	Q	L	L		L
Mexico	L	Q	L	L	Q	L
Netherlands	L	Q	L			L
New Zealand	L	Q	L			L
Norway	L	Q	m	L		m
Poland	L		L	L		L
Portugal			L			L
Slovak Republic	L	Q	L	L	Q	L
Slovenia			L	L		L
Spain	L	Q	L	L		L
Sweden	L	Q	L	L		L
Switzerland	L	Q	L	L	Q	L
Turkey		L	L			L
United Kingdom	L	Q	L		Q	L
United States	L		L	L		L
Partners						
Albania						
Argentina	L		L	L		L
Brazil		Q	L	L		L
Bulgaria		Q	L		Q	L
Colombia	L	Q	L	L	Q	L
Costa Rica	L	Q	L	L	Q	L
Croatia		Q	L			L
Hong Kong-China			L			L
Indonesia			L		Q	L
Jordan	L	Q	L			L
Kazakhstan			L		Q	L
Latvia	L		L			L
Liechtenstein			L			
Lithuania	L	Q	L	L		L
Macao-China	L	Q	L			L
Malaysia	L	Q	L			L
Montenegro	L	Q	L			L
Peru	L	Q	L	L	Q	L
Qatar	L	Q	L	L	Q	L
Romania	L	Q	L	L	Q	L
Russian Federation	L		L			L
Serbia	L	Q	L			L
Shanghai-China	L	L	L	L	L	L
Singapore	L	Q	L	L		L
Chinese Taipei		Q	L			L
Thailand	L	Q	L	L	Q	L
Tunisia	L		L	L	L	L
United Arab Emirates			L	L	Q	L
Uruguay	L	L	L		Q	L
Viet Nam			L			L

Note: "L" and "Q" show a statistically significant relationship between the opportunity to learn variables and mathematics performance. "L" when the relationship is linear and "Q" when it is quadratic.

Note: Table from Chapter 3, "Measuring Opportunities to Learn Mathematics," Figure I.3.3. (OECD, 2013).

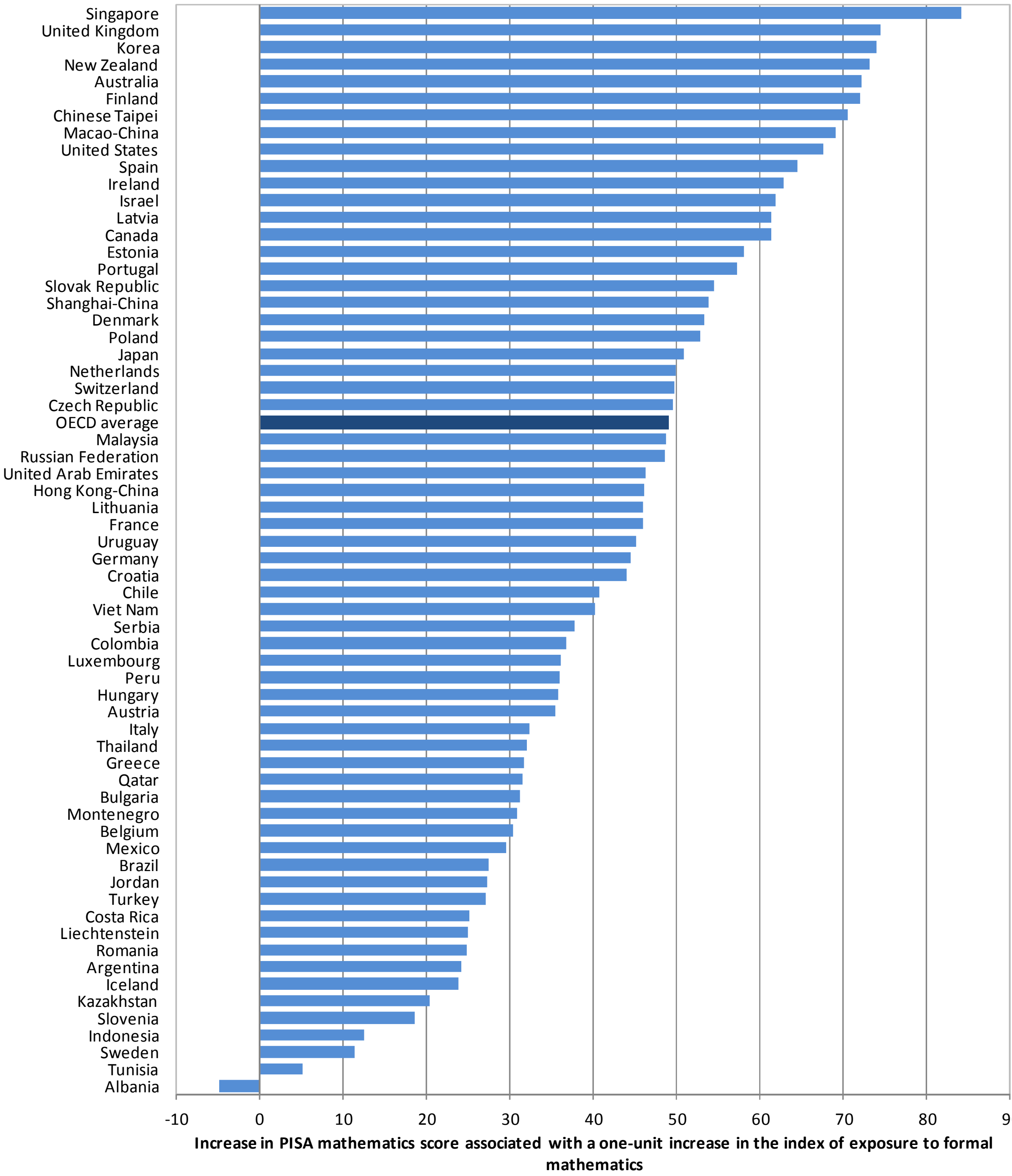
Figure 5. Relationship Between the Index of Exposure to Word Problems and Students' Mathematics Performance



Note: For the index of exposure to word problems the estimates come from a linear regression, positive values thus signal that greater exposure is more strongly associated with students' mathematics performance. Countries and economies are ranked in descending order of the strength of the relationship between the index of exposure to word problems and mathematics performance.

Note: Table from Chapter 3, "Measuring Opportunities to Learn Mathematics," Figure I.3.4a. (OECD, 2013).

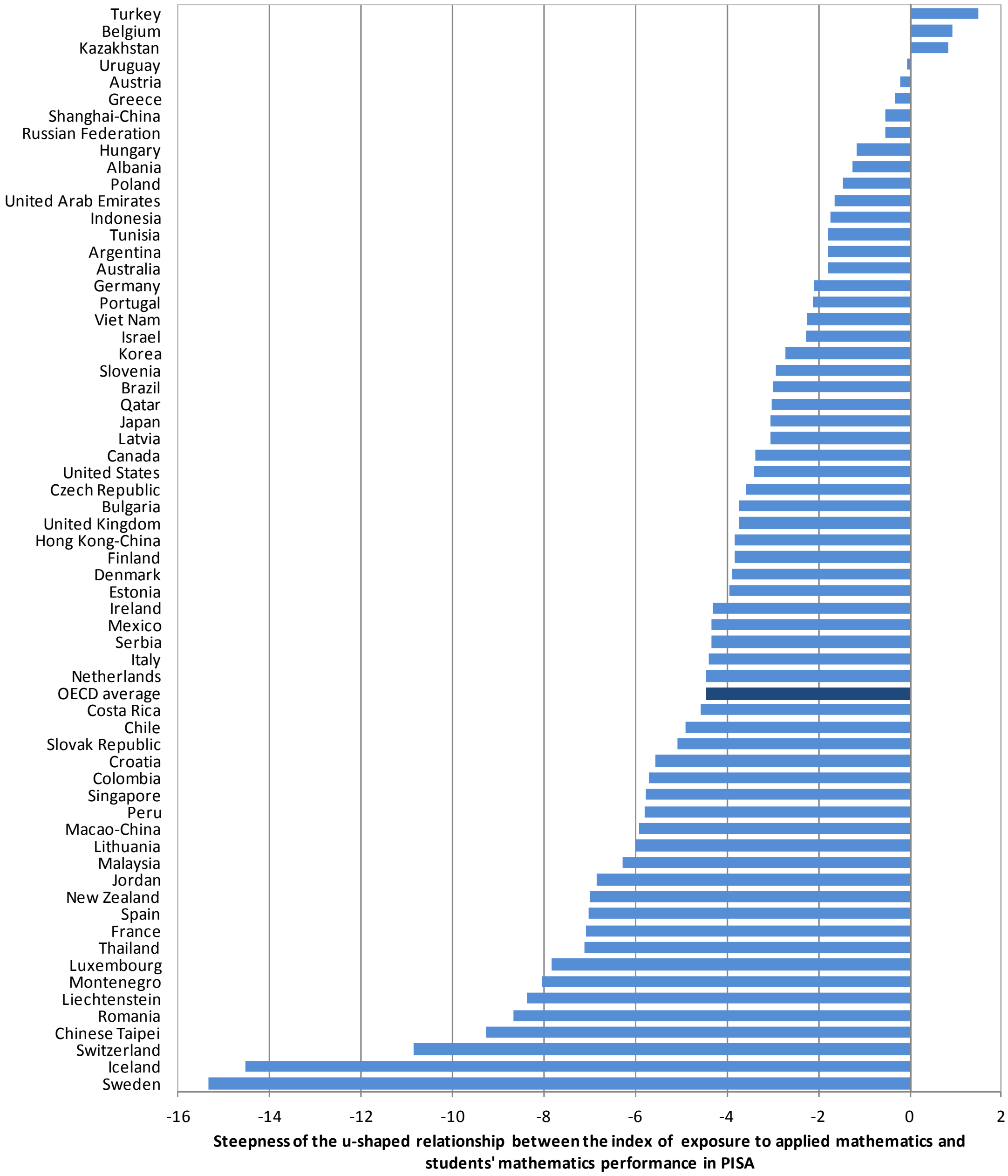
Figure 6. Relationship Between the Index of Exposure to Formal Mathematics and Students' Mathematics Performance



Note: For the index of exposure to formal mathematics the estimates come from a linear regression, positive values thus signal that greater exposure is more strongly associated with students' mathematics performance. Countries and economies are ranked in descending order of the strength of the relationship between the index of exposure to formal mathematics and mathematics performance.

Note: Table from Chapter 3, "Measuring Opportunities to Learn Mathematics," Figure I.3.4b. (OECD, 2013).

Figure 7. Relationship Between the Index of Exposure to Applied Mathematics and Students' Mathematics Performance.



Note: For the index of applied mathematics the estimates are from a regression with a quadratic term, meaning that negative values indicated an inverted u-shape relationship between the index and students' mathematics performance. Lower negative numbers point to steeper inverted u-shaped relationships. Countries and economies are ranked in descending order of the strength of the relationship between the index of exposure to applied mathematics and mathematics performance.

Note: Table from Chapter 3, "Measuring Opportunities to Learn Mathematics," Figure I.3.4c. (OECD, 2013).

Figure 8. The Quadratic Relationship Between Mathematics Performance and Students' Exposure to Applied Mathematics

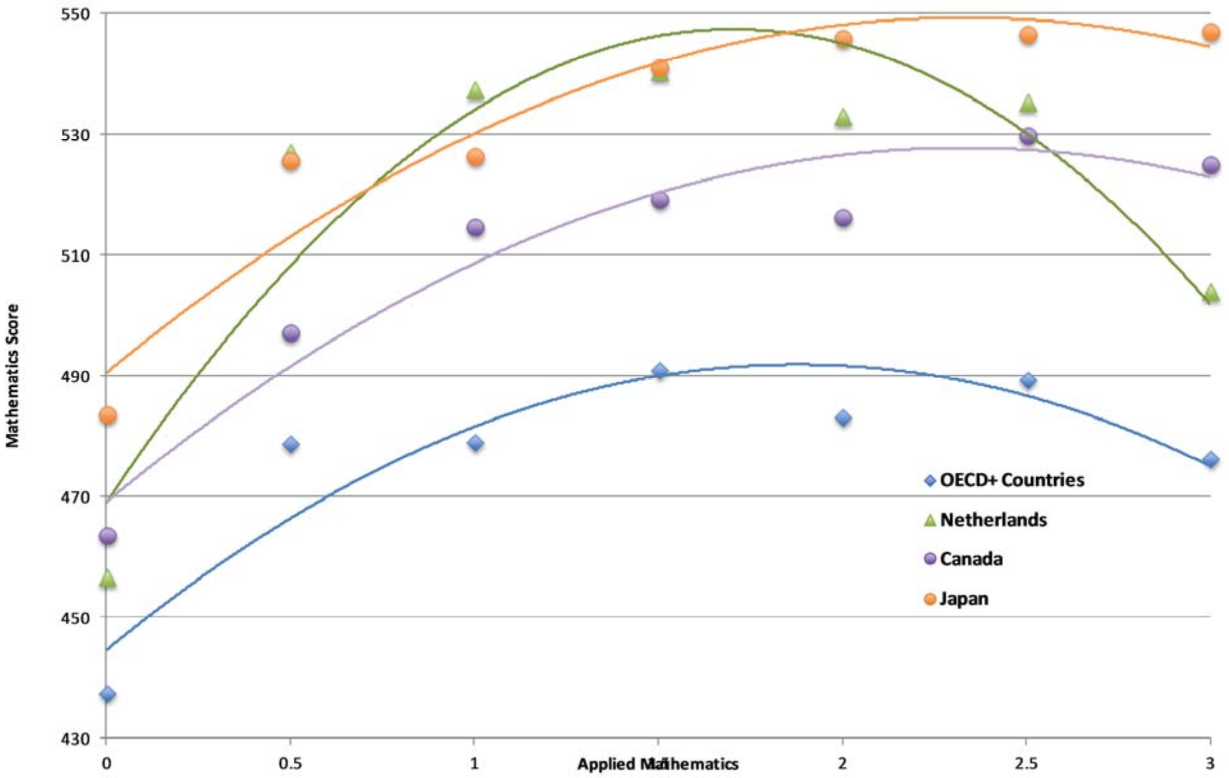


Table 4 summarizes the applied mathematics relationship to performance by identifying which of the 64 countries fall into which of four groups defined by whether the relationship was statistically significant or not at each of the student and school levels. This summary does not include formal mathematics, as it was significant in all but one country.

Table 4. Significance of Applied Mathematics (Linear or Quadratic) for Each Country in the Student and School Level Regressions

		School	
		Significant	*Not significant*
Student	***Significant***	Uruguay, United Kingdom, Finland, Slovak Republic, Thailand, Canada, Ireland, Bulgaria, Chile, Denmark, Peru, Costa Rica, Switzerland, Iceland, Qatar, Colombia, Mexico, Romania, Italy, Shanghai-China	Brazil, New Zealand, Croatia, Luxembourg, Lithuania, Chinese Taipei, France, Japan, Turkey, Sweden, Jordan, Macao-China, Netherlands, Spain, Montenegro, Singapore, Norway, Malaysia, Serbia
	Not significant	Estonia, Austria, Israel, United Arab Emirates, Indonesia, Kazakhstan, Tunisia	United States, Poland, Hong Kong-China, Greece, Albania, Latvia, Germany, Czech Republic, Hungary, Australia, Belgium, Argentina, Slovenia, Portugal, Liechtenstein, Korea, Russian Federation, Viet Nam

Note: Table from Chapter 3, "Measuring Opportunities to Learn Mathematics," Figure I.3.5. (OECD, 2013).

The nature of the overall relationship between applied mathematics and PISA 2012 mathematics literacy performance averaged over the OECD+ countries is presented in Figure 8. Three countries – Japan, Canada and the Netherlands – are also included to illustrate two forms of the quadratic relationship. The relationship Japan and Canada is more one of "leveling off" with only a slight decline in average performance at the highest level of exposure to applied mathematics while the Netherlands, illustrates a sharp average decrease in performance after a certain amount of exposure. The following quote explains this type of relationship in depth:

> *Among OECD countries, student performance is higher by about 40 points as the frequency of the encounters increased from "never" to "rarely"; but at a point between "rarely" and "sometimes" student performance reached a peak after which more frequent encounters with such problems had a negative relationship to performance. Fifteen-year-olds who frequently encounter applied problems scored about ten PISA score points below students who sometimes encounter such problems. (OECD, 2013)*

All of the previous results combined suggest a strong relationship between schooling, defined by the three OTL variables, and students' mathematics literacy performance. This relationship was apparent at both the individual student level as well as the school-within-country level. In the previous section this relationship was explored at the country level with the four PISA mathematics literacy sub-scales, the three PISA process scales and the computer-based mathematics literacy score. Table 5 summarizes the results of the two-level within country analyses with the three OTL indices and these various student performance scores. The numbers in the table indicate the percent of the OECD+ countries in which each OTL index was significant at the school and or the student level. Although significant relationships were apparent at both the individual student and school level more significant relationship were seen at the student level.

There is a striking similarity between the number of countries in which OTL is related to mathematics literacy as measured in the paper-and-pencil assessment and the computer-based assessment at both the individual student and school level. This suggests that within countries schooling is related to performance on literacy whether it is assessed by paper and pencil or by computer and that those relationships hold for the vast majority of countries – upwards of 70%.

Table 5. Percentage of all PISA Countries/Economies Exhibiting Significance for the OTL Variables in the Two-Level Within-Country Regressions for Each Score

	Math Literacy	*Change*	Quantity	*Shapes & Space*	Data	*Employ*	Formulate	*Interpret*	Computer Math	*Computer Problem Solving*
Student Level										
Word Problems	66	*67*	66	*59*	69	*70*	69	*64*	66	*58*
App. Math - Quadratic	50	*48*	50	*44*	55	*48*	52	*52*	50	*42*
Formal Math	98	*98*	97	*100*	95	*98*	97	*97*	98	*100*
School Level										
Word Problems	61	*61*	64	*50*	63	*61*	59	*56*	61	*48*
App. Math - Quadratic	39	*33*	39	*30*	36	*31*	28	*42*	39	*19*
Formal Math	95	*95*	95	*97*	97	*97*	97	*95*	95	*100*
Either Level										
Word Problems	77	*78*	78	*69*	84	*78*	78	*77*	77	*71*
App. Math - Quadratic	69	*67*	64	*53*	70	*63*	64	*72*	69	*55*
Formal Math	98	*98*	97	*100*	98	*98*	98	*98*	98	*100*

Variation in OTL – Issues of Equality of Opportunity to Learn

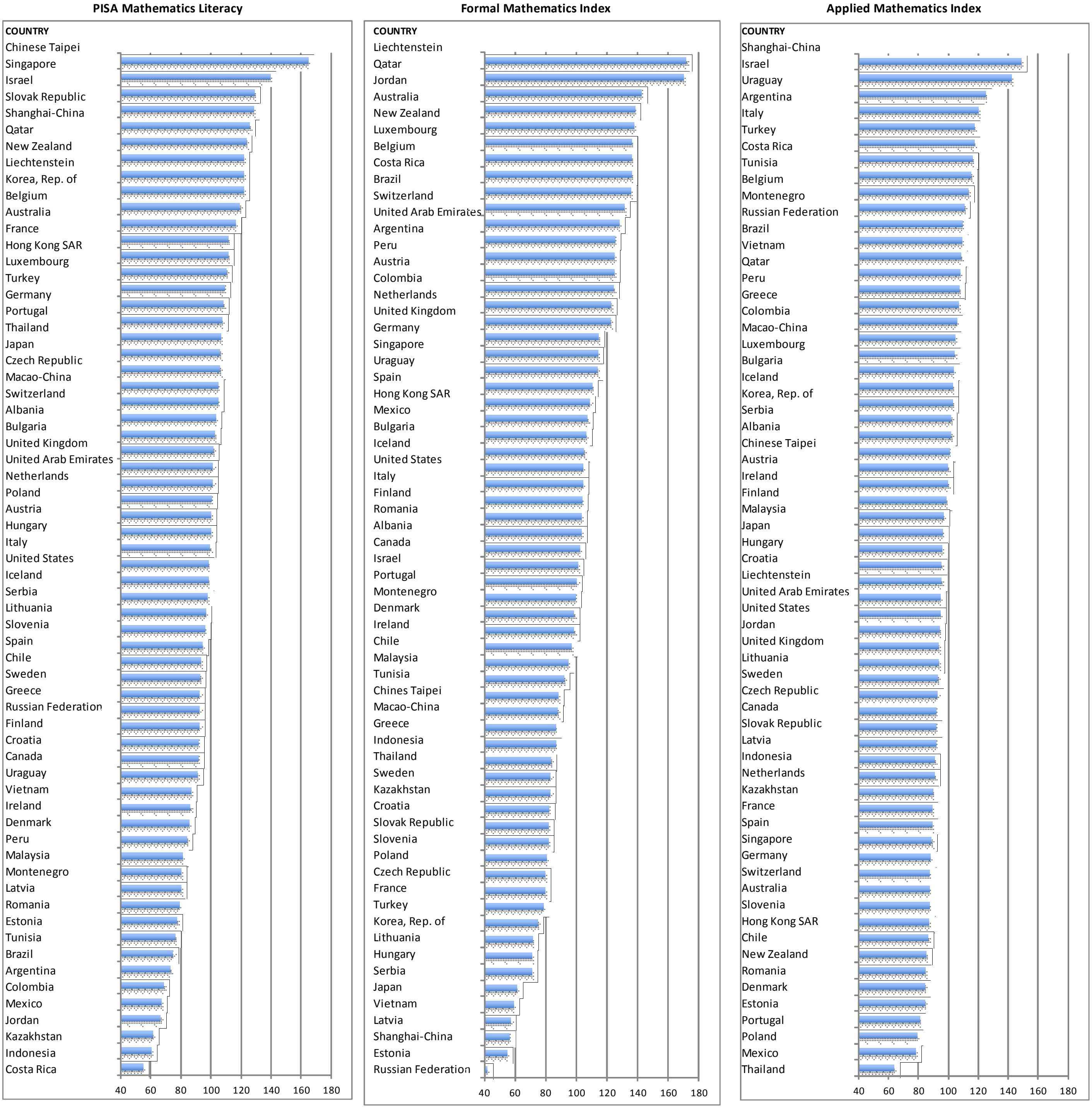
The issue of equality of opportunity to learn is a major policy issue in most countries. Schooling is viewed as a great equalizer – as a way for children of poverty to acquire the requisite knowledge to escape and achieve a better position in society economically, through better employment, as well as politically or socially as a well-equipped and informed citizen. Consequently, issues of equality in schooling, as may be revealed through variation in the three OTL indicators, are key issues to examine.

In this section within country variation in the OTL indices is explored along with the source of this variation, *i.e.*, whether variation occurs primarily at the student level or varies considerably from one type of school to another. The previous two sections have demonstrated that there are significant relationships between the three OTL indices and PISA mathematics literacy performance so variation in these OTL indices is not inconsequential. Therefore, if a country demonstrates large variation in the frequency with which students have encountered formal or applied mathematics this implies that some students have been placed at a disadvantage in terms of their exposure to opportunities to learn important/relevant content and may well be at risk for not obtaining the desired level of mathematics literacy. Clearly other components of the students' lives, such as the SES of their families, are relevant and can influence performance yet the analyses presented previously suggest that OTL is also important. This is particularly cogent from a policy perspective as OTL exposure and its variation creating inequalities is something amenable to policy focus and emphasis far more than students' economic or social characteristics.

As such the central measure of inequality, the total variance in formal and applied mathematics OTL was examined. Figure 9 summarizes the total variation across all students within a country in both measures of OTL. So as to make them more readily comparable the total variance of a country is expressed as a ratio with respect to the average variance over the OECD+ countries, which was assigned the value of 100.

Among the OECD+ countries: Belgium, Australia, New Zealand and Luxembourg demonstrated the greatest variation in formal mathematics exposure. For those countries the total variation in formal mathematics was around one-third again larger than the OECD+ average. On the other end of the spectrum with the least amount of total variation are Estonia and the Russian Federation where the variance is about one-half that of the OECD+ average.

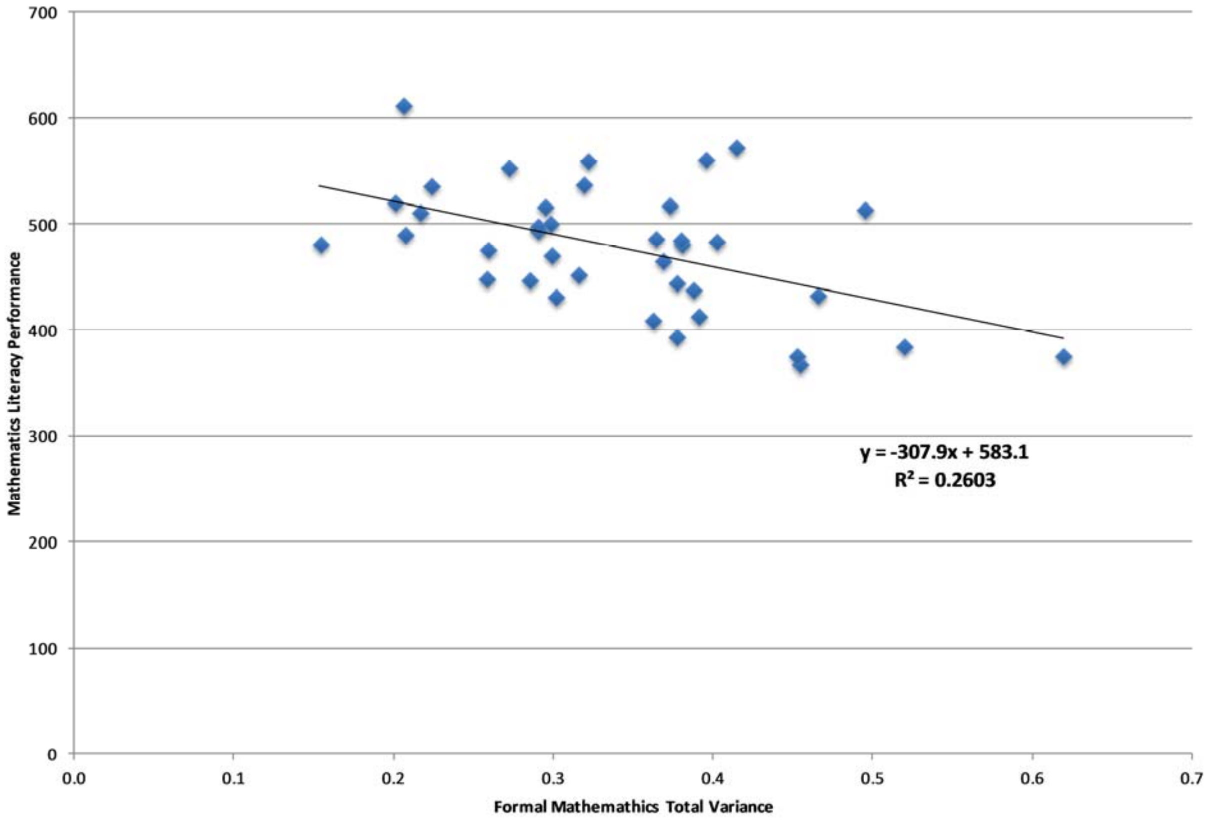
Figure 9. Size and Rank of Total Within-Country Variance for Mathematics Score, Formal Mathematics OTL and Applied Mathematics OTL



Such large differences among the countries in terms of this ratio suggest that inequality of opportunity for formal mathematics could well be related to country differences in performance. Since opportunity to learn formal mathematics was shown to be related to performance on the literacy test within countries, large variation in opportunity could be related to greater variation in performance which could impact the overall country average. This would especially be the case if the larger variance resulted from less exposure to formal mathematics for a relatively large number of students. In that way, variation over countries in the size of OTL variance could then suggest a cross-country relationship between within-country variance in OTL and mean performance. But if the average amount of OTL in a country was small to begin with, then larger variation might not be related to performance since the low amount of average OTL and the variation around it would have a relatively small influence on performance.

To explore this issue we split the countries into two groups based on the amount of OTL related to formal mathematics; those above the OECD+ average and those below it. For each set of countries, a linear regression was done relating the total country variance for formal mathematics with the average student performance. For the set of countries with below average opportunity related to formal mathematics, there was no statistically significant relationship with performance. However, among the set of countries with OTL in formal mathematics above the OECD+ mean, there was a statistically significant relationship between within-country variation in formal mathematics and performance. This relationship can be seen in Figure 10. The negative relationship across countries implies that less variation in OTL at the country level is associated with higher overall average performance on the literacy test.

Figure 10. Literacy Performance Related to the Total Within-Country Variance for Formal Mathematics for Countries Above the OECD+ Formal Mathematics Mean



The total variation in OTL as defined by applied mathematics is also represented in Figure 9. The OECD+ countries with the largest variation in applied mathematics were Italy, Turkey and Argentina, while those with the smallest variance were Portugal, Poland, Mexico and Thailand. There was no statistically significant relationship of the total within-country variance to average country performance.

In considering the policy implications related to variation in OTL, it is necessary to understand the sources of such variation. Variance components provide an essential vehicle by which to analyze the sources of variation in the OTL variables. To explore this issue further, a variance decomposition was done to estimate the percentage of variation related to differences among schools versus the percentage of variation within schools. Each of these components as contributing to the total variation has policy implications related to ameliorating inequalities.

Between school variation typically comes from two major sources of variation – SES differences stemming from the socioeconomic differences of the students that attend the schools and different types of schools such as those intended to prepare students for tertiary education or those intended to prepare students to immediately enter the job market. These different types of schools often have entrance requirements that are quite different from each other. Within-school variation can stem from variation due to differences among teachers teaching the same course, differences in the courses students have taken, as well as within-school tracking of students into different programs similar to the different types of schools mentioned above.

Table 6 lists the percentage of the total variance attributable to within school variation for the two OTL indices. The percentage of the total variation in formal mathematics attributable to within-school differences averaged over the 41 OECD+ countries was 81%. In no OECD+ country was this percentage less than 50%. There was, however, variation across countries with values ranging from 57% to 93%. In Austria only 57% of the total variation in formal mathematics was attributable to within-school differences while the comparable figures for the United States, Poland, Portugal, Estonia, Greece, Iceland, Sweden and Ireland were all 90% or more. These figures must be placed, however, in the context of the size of the total variance in formal mathematics (see Figure 9).

Table 6. Country Means for Various Statistical Indices Related to Issues of Inequality

Country	Average student performance in mathematics	Strength of the relationship between student performance and socio-economic status [1]	Mean index of exposure to formal mathematics	Variation of the index of formal mathematics as a proportion of the OECD variance	Variation of the index of formal mathematics as a proportion of the <u>OECD+</u> variance	<u>INDEX</u> of exposure to <u>APPLIED</u> mathematics	Variation of the index of <u>APPLIED</u> mathematics as a proportion of the <u>OECD+</u> variance	Within-school variation of the index of exposure to formal mathematics as a proportion of the sum of the within- and between-school variation	Within-school variation in socio-economic status as a proportion of the sum of the within- and between-school variation [2]	Within-school variation of student performance as a proportion of the sum of the within- and between-school variation [3]
		Percentage of explained variation in student performance								
Australia	504	12.3	1.7	134	141	2.0	91	80	77	72
Austria	506	15.8	1.5	129	126	1.8	104	57	71	52
Belgium	515	15.0	1.8	141	140	1.9	118	72	72	50
Canada	518	9.4	2.0	100	105	2.1	96	89	83	80
Chile	423	23.1	1.7	92	99	2.1	90	75	47	57
Czech Republic	499	16.2	1.8	78	82	1.6	96	71	76	49
Denmark	500	16.5	1.6	98	100	2.0	88	88	82	84
Estonia	521	8.6	2.0	56	56	1.8	87	92	81	83
Finland	519	9.4	1.7	96	106	1.7	102	88	91	92
France	495	22.5	1.9	87	84	2.0	93	w	w	w
Germany	514	16.9	1.7	118	116	2.0	91	67	74	47
Greece	453	15.5	1.9	92	89	1.9	111	93	73	68
Hungary	477	23.1	2.0	80	73	1.9	99	72	63	38
Iceland	493	7.7	1.1	105	107	2.0	107	96	86	90
Ireland	501	14.6	1.5	100	100	1.9	103	91	80	82
Israel	466	17.2	1.8	111	103	1.8	148	80	75	58
Italy	485	10.1	1.8	107	107	1.8	122	68	76	49
Japan	536	9.8	2.1	61	63	1.7	100	72	78	47
Korea, Rep. of	554	10.1	2.1	74	76	1.8	107	74	78	60
Luxembourg	490	18.3	1.4	138	138	1.9	108	86	74	61
Mexico	413	10.4	1.8	117	110	2.2	81	82	57	65
Netherlands	523	11.5	1.5	123	125	2.1	94	68	82	34
New Zealand	500	18.4	1.5	139	140	2.0	88	83	78	76
Poland	518	16.6	1.8	82	83	2.0	83	92	76	79
Portugal	487	19.6	1.7	100	103	2.2	84	90	69	70
Slovak Republic	482	24.6	1.7	86	85	1.9	95	67	64	50
Slovenia	501	15.6	1.9	86	85	1.9	91	79	75	41
Spain	484	15.8	1.9	119	114	2.0	93	88	75	81
Sweden	478	10.6	0.8	86	84	1.7	97	92	87	87
Switzerland	531	12.8	1.4	137	133	1.9	91	60	83	64
Turkey	448	14.5	1.9	83	82	2.0	122	85	72	38
United Kingdom	494	12.5	1.6	118	124	1.9	97	82	79	72
United States	481	14.8	2.0	113	109	2.0	98	90	74	76
OECD Average	494	14.6	1.7	100	100	1.9	100	80	76	64
Argentina	*388*	*15.1*	*1.4*	*131*	*129*	*1.9*	*124*	*75*	*67*	*56*
Brazil	*391*	*15.7*	*1.4*	*139*	*138*	*2.0*	*113*	*72*	*63*	*57*
Chinese Taipei	*560*	*17.9*	*2.0*	*89*	*91*	*1.8*	*105*	*82*	*77*	*58*
Colombia	*376*	*15.4*	*1.7*	*138*	*128*	*2.2*	*110*	*84*	*63*	*65*
Indonesia	*375*	*9.6*	*1.6*	*91*	*88*	*2.3*	*94*	*82*	*63*	*48*
Malaysia	*421*	*13.4*	*1.6*	*99*	*97*	*2.0*	*100*	*88*	*72*	*68*
Russian Federation	*482*	*11.4*	*2.1*	*45*	*44*	*2.0*	*114*	*95*	*75*	*73*
Thailand	*427*	*9.9*	*1.7*	*79*	*85*	*2.4*	*67*	*85*	*62*	*58*
OECD+ Average	*482*	*14.6*	*1.7*	*100*	*100*	*2.0*	*100*	*81*	*74*	*63*
Albania	394	m	2.1	114	110	2.2	106	93	m	95
Bulgaria	439	22.3	2.0	122	113	1.9	108	82	60	47
Costa Rica	407	18.9	1.5	141	138	1.7	121	79	62	58
Croatia	471	12.0	2.1	87	85	1.8	99	88	76	56
Hong Kong SAR	561	7.5	1.8	111	111	1.8	90	93	68	58
Jordan	386	8.4	2.1	156	150	2.2	98	85	80	64
Kazakhstan	432	8.0	2.0	86	86	2.2	93	91	77	63
Latvia	491	14.7	2.0	61	59	1.8	95	89	75	74
Liechtenstein	535	7.6	1.5	156	174	2.0	99	54	86	37
Lithuania	479	13.8	1.6	73	74	1.9	97	92	79	69
Macao-China	538	2.6	2.2	88	90	1.6	109	86	74	58
Montenegro	410	12.7	1.9	109	106	1.9	115	93	81	64
Peru	368	23.4	1.8	140	129	2.1	112	80	54	54
Qatar	376	5.6	1.7	182	176	2.0	112	76	75	54
Romania	445	19.3	2.0	108	107	2.1	88	78	64	55
Serbia	449	11.7	2.0	80	77	1.8	106	89	78	54
Shanghai-China	613	15.1	2.3	57	57	1.6	155	83	67	53
Singapore	573	14.4	2.2	113	116	2.0	92	83	76	63
Tunisia	388	12.4	1.2	98	93	2.1	120	94	67	51
United Arab Emirates	434	9.8	2.1	137	132	2.1	99	80	74	56
Uraguay	409	22.8	1.6	127	119	1.7	130	76	60	58
Vietnam	511	14.6	2.0	60	61	1.7	113	83	58	48

Note: This is a modified version of a table appearing in Volume II, PISA 2012.

The average percentage of the variance attributable to within-school differences for the OECD$^+$ countries was 96% for both word problems and applied mathematics with essentially no variation across the countries – all countries had a percentage of 90% or more. For this reason these percentages are not reported in Table 6. Clearly the majority of the total variation in the three OTL indices was found to be at the within-school level pointing primarily to differences in course content, variation in what teachers covered, or the existence of within school tracking of students.

Table 6 also lists the percentage of total variation in student performance that is attributable to within-school variation. The average for the OECD+ countries was 63% and the variations ranged from 34% in the Netherlands to 92% in Finland. Not surprisingly for the 3 OTL variables, SES, and student performance most of the variation in most of the countries is at the within-school level but there are some notable differences especially with respect to student performance such as in Hungary, Turkey and The Netherlands.

SES and OTL and How They Relate to Performance

From a previous section we know that OTL is related to student performance at the within country level where the estimated average effect size for the OECD+ countries was around one-half a standard deviation. In Table 6 a measure of the strength of the relationship between student performance and SES is given for each country. It provides an estimate of the percentage of variation in performance that is accounted for by SES. For all the OECD+ countries that relationship accounts for less than 25% of the variance leaving a substantial amount of the variance which could be related to schooling as measured by the three OTL variables. In fact, for 7 countries SES accounted for less than 10 percent of the performance variation.

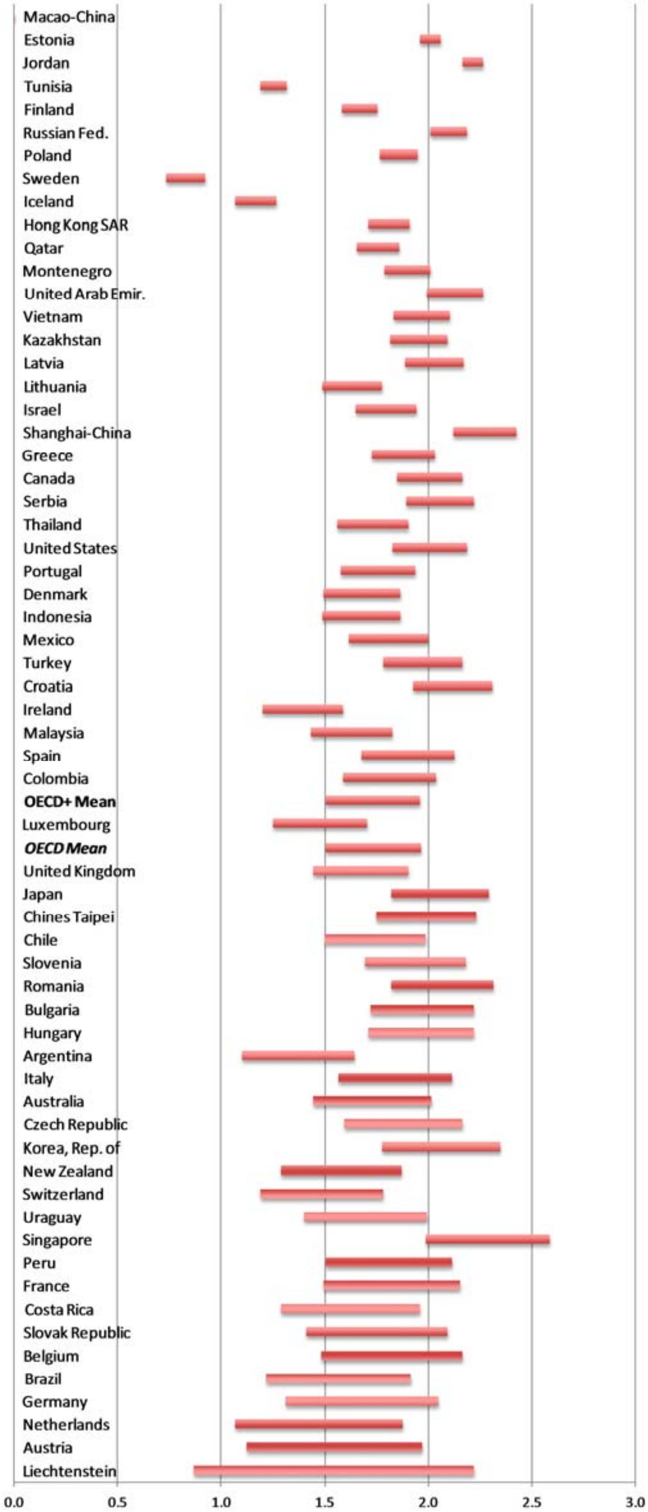
Using student data from all countries we fitted a model including SES and OTL at all three levels: student, school, and country and found statistically significant relationships for both SES and OTL. The impact of SES was to lessen the magnitude of the relationship of OTL to student performance but the relationship of OTL was never not statistically significant. The issue for this section of the paper is to examine the relationship of SES to OTL but also to study how that relationship impacts each of their relationships to performance. If such a relationship exists between SES and OTL it raises the possibility of an even greater impact of SES on performance as it not only has a direct relationship to student performance but also an indirect one through its relationship to OTL which itself is related to student performance.

PISA has classified schools within each country as advantaged and disadvantaged depending on the SES composition of the students within that school. Such a definition has the advantage of defining SES composition within the context of a particular country. This avoids the problems of a general SES index that is supposed to scale across countries. In this case, the schools are classified only according to the distribution of SES within each country specifically, thus making the distinction between advantaged and disadvantaged relative in a country specific fashion. The differences between the means of two groups of schools on the two main OTL variables – formal and applied mathematics – were calculated and these were then averaged over schools within a country to estimate the mean difference between advantaged and disadvantaged schools for that country.

The average difference for the OECD+ countries was approximately a half a point on the formal mathematics index whose scale ranged from 0 to 3 (see Figure 11). The differences were statistically significant in all countries except two, with the estimated differences ranging from .1 to .8. Macao-China was the one place where there was literally no difference in means between the advantaged and disadvantaged schools while the largest differences were found in the Netherlands, Brazil, the Slovak Republic, France, Germany and Austria. The smallest differences were found in the Russian Federation, Hong Kong-China, Sweden, Poland, Iceland, Finland and Estonia. For applied mathematics the average difference for the OECD+ countries was smaller than was the case for formal mathematics (-.03) and significant differences were found in only about half of the countries. The largest difference favoring the advantaged schools was .26 which occurred in Belgium, but in Russia, Poland, the Netherlands, Israel, Shanghai-China and Greece there were statistically significant differences favoring the disadvantaged schools.

To illustrate more concretely the differences in OTL between the two types of schools, the difference in terms of exposure to the topic of linear equations was calculated. This was one of the topics making up the formal mathematics index. The OECD+ average percent of students in a country that indicated encountering this topic frequently or more than frequently was 63%. But, averaged over the same 41 countries, the percent for those students attending an advantaged school was 75% compared to 53% for those attending a disadvantaged school – a difference of about 20%. The differences across countries ranged from more than a 50 percent point difference to less than a ten percent difference.

Figure 11. Additional Exposure to Formal Mathematics of Advantaged Schools Compared to Disadvantaged Schools Within Each Country



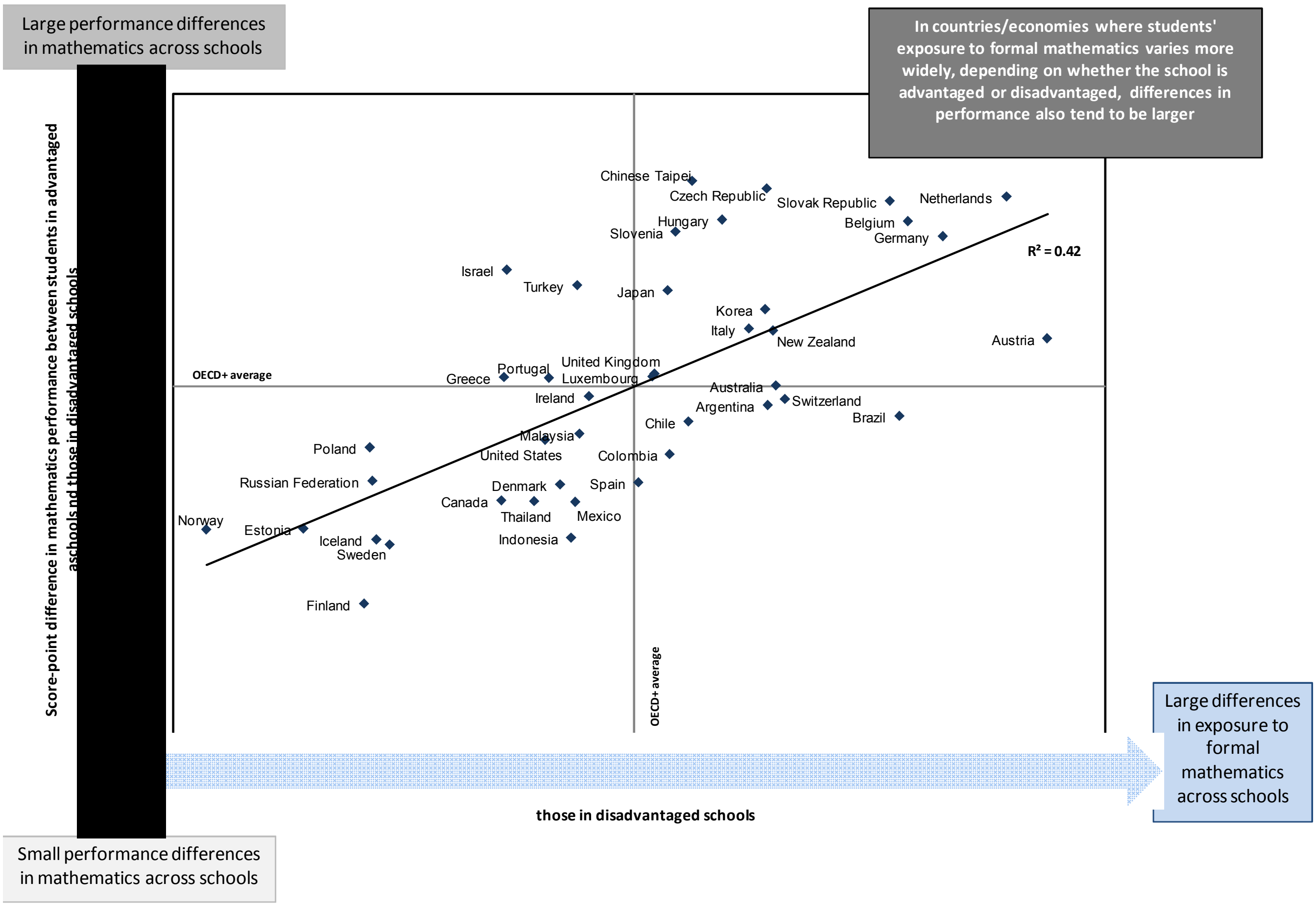
The question we now turn to is, does the gap in performance related to the two types of schools as defined by SES relate to the corresponding gap in OTL as defined by formal mathematics for the OECD+

countries. To address this question the difference between the advantaged and disadvantaged schools within a country in student performance was calculated for each country. Figure 12 shows the relationship between the average difference between the two types of schools in student performance within a country and the corresponding average difference in the coverage of formal mathematics across the 41 OECD+ countries. This relationship was statistically significant accounting for 49% of the variance in performance across countries.

The nature of the relationship suggests that a one-point difference in the average amount of coverage in formal mathematics between the advantaged and disadvantaged schools within a country predicts a 123 point difference in average performance between those same schools. Put simply, in the OECD+ countries where there is a difference of one point in the coverage of formal mathematics favoring the advantaged schools, the gap in performance is predicted to be more than a full standard deviation. This implies that SES has an indirect relationship to student performance through it relationship to the coverage of formal mathematics which, as shown in the previous section, also has a significant relationship to performance. This indirect relationship is in addition to the direct relationship that SES has to performance.

The same analysis was done for applied mathematics but there was not a statistically significant relationship between the gap in applied mathematics OTL and the gap in performance across the OECD+ countries.

Figure 12. Magnitude of Performance Differences Related to Students' Exposure to Formal Mathematics, by Schools' Socio-economic Profile: OECD+ Countries



Two other analyses were done to further study the relationship of SES and OTL to each other and to student performance. First, a two-level model was fitted within each country relating SES to student performance at both the student and school level providing estimated regression coefficients that indicated for a one point difference in SES what the predicted difference in performance would be. The same model was again fitted but with one additional variable – OTL related to formal mathematics again at both the student and school level. The difference in the two regression coefficients relating SES to performance – one with and one without controlling for OTL - estimates the reduction in the relationship of SES to performance when formal mathematics is taken into account.

The difference between the two coefficients implies that the average change in the predicted performance at the student level for the OECD+ countries was seven points for a one point change in the SES index controlling for OTL. At the school level there was a 31 point change in the relationship of SES to performance. The range across the 41 OECD+ countries was large both at the within school and the between school levels but especially so at the school level where the change in performance for a one point difference in the PISA international SES index ranged from 118 in the Netherlands to 12 in Poland, Thailand and Ireland. Korea and Japan also had sizeable differences in average performance on the literacy test predicted from the differences in the SES index. At the within-school level the range in differences on performance was much smaller varying between 15 for New Zealand to 3 for Greece and Colombia.

These results are consistent with those found previously reinforcing the notion that within most countries SES has both a direct and an indirect relationship to performance. The indirect relationship occurs where SES at either the individual student level or at the school level is related to opportunity to learn, implying that students or schools at different levels of SES receive different amounts of formal mathematics in their schooling. Formal mathematics was already found to be significantly related to performance in all of the OECD+ countries.

At the policy level the implications are clear, the larger the reduction in the predicted performance (as evidenced by the difference between the two regression coefficients) the larger the impact that OTL has in eliminating the indirect relationship and in that sense lessening the overall role that SES plays in that country. The greater the reduction the more successful a country has been in overcoming the challenge that SES represents to student performance.

We started this section looking at variance in OTL as an indication of the degree of inequality in schooling especially at the school level; we now return to that issue. Given the important role that public policy plays with respect to the structure of schools around issues of equality of opportunity, the focus of this analysis centers on school level variation in both OTL and SES and their relationship to school performance. The three indices used were: the percent of the total country variance attributable to between school differences for formal mathematics, SES, and performance. The PISA SES index of economic, social and cultural status was related to performance. The results show a statistically significant relationship with an estimated R^2 equal to .26 indicating that in countries in which there is a relatively small amount of between school variation, there is also a relatively small amount of variation in performance across those schools. Concerning schools, it must be kept in mind that relatively low or high variation as referenced in this set of analyses does not refer to the magnitude of the variance at the school level for a country, but to the relative amount of total variance that is between schools.

More interestingly is the fact that the parallel analysis of the extent of the coverage of formal mathematic as related to performance yields a stronger relationship as evidenced by an estimated $R^2 = .52$. The nature of the relationship is the same in that in those countries where the relative amount of variation among schools in exposure to formal mathematics is small, so is the relative amount of variation in performance. The R^2 from this analysis is twice that of the SES to performance analysis implying that at the between-school level in countries with a more tracked, less uniform approach to the distribution of

OTL in formal mathematics across schools, there will correspondingly be a greater proportion of the country variance in performance at the school level as well.

Countries in which secondary schools are tracked as to who can attend them usually in terms of ability would be predicted based on this model to have relatively greater variation at the school level. The same general relationship is true of schools segregated by SES which is typically related to housing patterns. Giver the larger R^2 for OTL, one policy implication is that having coverage of formal mathematics for 15-year olds be fairly uniform across schools within a country would be one way to reduce the relative amount of total variation in performance related to differences among schools which is often an important policy goal of governments related to ensuring equity.

Relating OTL to Some Non-cognitive Outcomes of Schooling

As a part of PISA students were asked to respond to questions describing their attitudes and beliefs about mathematics. These are considered by many policy makers as important additional outcomes of schooling based on the belief that the goal of schooling with respect to mathematics is not only related to student learning but also to such issues as developing an interest in mathematics or developing a confidence to do mathematics as well as others. Four PISA belief scales were analyzed to determine if OTL also has a relationship to some of the non-cognitive outcomes associated with schooling.

These scales were: 1) an openness to problem solving; 2) anxiety about mathematics; 3) interest in mathematics; and the belief that "I am not good at mathematics" (see OECD, 2013b for a definition of these four scales from the student questionnaire).

Table 7 summarizes the results of the four analyses by indicating the number of countries for which a statistically significant result was found at each level relating each of the 3 OTL variables to the four belief scales. In general, formal mathematics was related to the four scales in almost all countries especially at the student level. Also presented in Table 7 are the regression coefficients for each of the OTL variables at both the between-school and within-school levels averaged over the 41 OECD+ countries. Formal mathematics was positively related to both: openness to solving complex problems and to a greater degree of interest in mathematics.

Table 7. Two-Level OTL Model Relating OTL to Student Beliefs and Motivation for Mathematics – Number of Countries with a Significant Coefficient for Each OTL Variable and the Estimated Average Regression Coefficients

	Student						School					
	Word problems		Applied mathematics		Formal Mathematics		Word problems		Applied mathematics		Formal Mathematics	
Percent of Countries with significant coefficients	(+)	(-)	(+)	(-)	(+)	(-)	(+)	(-)	(+)	(-)	(+)	**(-)**
Attributions to Failure - Not Good at Maths Problems	7	4	16	1	0	61	9	1	6	3	0	38
Openness for Problem Solving - Like to Solve Complex Problems	30	0	33	0	62	0	8	3	4	0	39	1
Mathematics Interest Scale	29	0	19	3	63	1	12	5	12	1	33	5
Mathematics Anxiety Scale	8	8	45	1	0	64	5	6	14	1	0	54
Average Coefficients												
Attributions to Failure - Not Good at Maths Problems	0.02	-0.02	0.05	-0.03		-0.26	0.08	-0.04	0.11	-0.09	0.05	-0.22
Openness for Problem Solving - Like to Solve Complex Problems	0.06	-0.02	0.10	-0.02	0.51		0.13	-0.11	0.18	-0.09	0.47	-0.01
Mathematics Interest Scale	0.05	-0.02	0.06	-0.03	0.41		0.09	-0.10	0.23	-0.09	0.36	-0.13
Mathematics Anxiety Scale	0.02	-0.04	0.09	-0.07		-0.43	0.09	-0.10	0.21	-0.12	0.08	-0.38

On the other hand, but in the same positive way, increased exposure to formal mathematics is negatively related to both the anxiety scale and to the belief that "I am not good at mathematics." This implies that students are less anxious about mathematics and more inclined to disagree with the statement, “I am not good at mathematics.” In this way the degree of exposure to formal mathematics is related in a positive way not only to performance on the literacy test, but also to desirable non-cognitive outcomes of schooling.

Results for applied mathematics and word problems were more varied across the four scales and mostly existed at the within-school and not the between-school level. Similar to formal mathematics, applied mathematics was positively related both to being open to do complex problems and having a greater degree of interest in mathematics. However, to both mathematics anxiety and agreement with not being good at mathematics the relationship was positive in contrast with that for formal mathematics. The implication was that on average a greater exposure to applied mathematics problems increased student anxiety and a greater inclination to believe they were not very good at mathematics. The latter is harder to understand unless the difficulty students have in doing applied problems as evidenced by the fact that the percent correct on most items in PISA are low, garners more self-doubt and anxiety – an interesting finding in need of further study.

What is clear from Table 7 is that opportunity as defined by the three scales was not only related to performance with respect to literacy as measured by a paper and pencil assessment and to the problem solving assessment but also to several of the non-cognitive outcomes of schooling. This includes interest, confidence, and anxiety as it relates to mathematics as well as openness to problem solving.

Concluding Thoughts

The research question identified at the beginning of this paper asked whether schooling was related to performance on the PISA 2012 mathematics literacy based assessment. The answer is a qualified yes – qualified by the nature of the data. As PISA 2012 represents cross-sectional data, this does not permit more sophisticated analyses that would support causal inferences. Nonetheless, opportunity to learn (OTL) was related to student performance on both versions of the literacy test and at all three levels – country, school and student. In addition, the two-level hierarchical analyses identified statistically significant relationships between the three OTL variables and performance in a majority of the 42 OECD+ countries.

The average estimated regression coefficients suggested an effect size of approximately a half of a standard deviation for formal mathematics and one-tenth of a standard deviation for encountering applied mathematics problems. Exposure to traditional word problems found in typical textbooks had an even smaller effect size. The term “effect size” is used here in the classical statistical sense that identifies the relative magnitude of the relationship between OTL and performance; it does not imply causality. The fact that the relationship exists across the two versions of the literacy test (paper/pencil versus computer) and the four subtests as well as the problem solving assessment and in each case within a majority of countries supports the importance of this finding.

The finding that those students who have more exposure to formal mathematics have a strong relationship to performance suggests that such OTL is related to performance whether the assessment is a more traditional curriculum defined assessment, as in TIMSS, or a literacy-defined assessment as in PISA. What is interesting here is the relationship of the frequency of exposure to applied mathematics problems having a relationship to performance on the literacy assessment even after controlling for the amount of exposure to formal mathematics. This suggests that in addition to the relationship of formal mathematics, those students who were given more frequent opportunities to do applied mathematics problems in school tended to do better on the literacy assessment than those who did not have such opportunities.

The complication with that statement is that it turns out in most countries the nature of the relationship of exposure to applied mathematics and performance is quadratic. This implies that more is not always better. For some countries the relationship levels out and more frequent exposure adds nothing beyond a certain point. In other countries, more such exposure is negatively related to performance beyond a certain amount. The point at which the relationship to performance levels out or turns negative is not well specified given the limited range of the scale employed in the study. These are particularly intriguing results that call for further research.

One of the contentious issues facing mathematics education is the role of real-world applications. Perhaps in a limited way, these results contribute to the dialogue at least in suggesting that more is not necessarily better, at least with respect to an assessment of mathematics literacy. In sum, the simple answer to the research question that guided this work is that PISA joins TIMSS in supporting the important conclusion that schooling does matter; the exposure to the formal mathematics of algebra and geometry taught worldwide to 15 year olds is not only related to performance on a content-specified test defined over the typical K-12 topic domain but also to an assessment similarly defined over that domain but focused more on the application of that mathematics to solve real world problems.

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