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ARTIGOS

Comparison of 6th-8th Graders' Efficiencies, Strategies and Representations Regarding Generalization Patterns

Estudo Comparativo da Eficácia de Estratégias e Representações Usadas por Estudantes da Escola Básica em Problemas Relativos à Generalização de Padrões

YaşarAkkan*

Abstract

The study aims to compare and determine 6th-8th graders' efficiencies, strategies and representations of student from different grades (from 6th to 8th) when dealing with problems related to linear and quadratic patterns. Research data was obtained from tests applied to 246 students and clinical interviews implemented with 18 students. It was shown that when grade increases students' efficiencies of generalizing pattern improve in a positive way in all levels. Besides this, as learning levels of 6th and 8th grade students increase, the variety in pattern generalization strategies changed at least in all types of patterns. While students at different learning levels generally used recursive strategy to solve all the problems, the number of the students using explicit strategies is relatively low. The students of high levels use many types of representation in generalization; in contrast, the weak level students prefer mostly numerical representations.

Keywords: Pattern. Strategies. Efficiencies. Representations. 6th- 8th Grades.

Resumo

Neste artigo são estudadas as estratégias e representações usadas por alunos turcos de diferentes séries (da 6ª a 8ª) quando resolvendo problemas que envolvem a busca de

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padrões lineares e quadráticos. Os dados foram coletados a partir da aplicação de testes específicos em 246 alunos e de entrevistas clínicas com 18 desses alunos. Os resultados apontam que a eficácia nos padrões de generalização aumentam positivamente de uma série escolar para outra, bem como aumenta a variedade das estratégias de generalização. Enquanto alunos de diferentes momentos da escolarização usam estratégias recursivas para resolver os problemas relativos à busca de padrões, mas a quantidade de estudantes que mobilizam estratégias explícitas é relativamente pequena. Alunos de séries mais avançadas usam vários tipos de representações no processo de generalizar, ao passo que alunos das séries mais iniciais, de modo acentuado, preferem a representação numérica.

Palavras-chave: Padrões. Estratégias. Eficácia. Representações. 6ª a 8ª series.

1 Introduction

In order to educate 21st century people the field of mathematics education is passing through dramatic changes. In that changing process, the approaches on high-level mathematical reasoning such as exploring, conjecturing and generalizing has taken a significant place in basic mathematics instruction (THOMPSON; THOMPSON, 1995). Thus, new standards and curricula have been developed and put into practice around the world. Turkish education system has affected by this process and Turkish elementary mathematics education curriculum has changed following the new expectations stated by Turkish Ministry of Education (TME). Efforts to study, recognize and develop patterns in new curriculum appear as an important learning field, based on the fact that

> Mathematics is the science of patterns and harmonies. In other words, mathematics is the science of number, shape, space, size and the relations between them. Discovering the relations that the patterns include, and generalizing them can be helpful for students in developing their skills while perceiving the world around them better. In addition, the patterns represented in different forms and especially expressed as symbolic will make agreat contribution to basic concepts of algebra to be formed [...] (TME, 2005, p.95*)*.

Many mathematics educators have dealt with patterns from different points of view and agreed on the idea that discovering and generalizing patterns are important for learning mathematics. They have also expressed that the study of patterns could improve students' algebraic concepts at early ages, and contribute to that algebraic thinking required in future learning (KENNY; SILVER, 1997; ENGLISH; WARREN, 1998; ORTON; ORTON, 1999; ZAZKIS; LILJEDAHL, 2002; LANNIN, 2003; SMITH, 2003). Lee (1996, p.103) stated that "algebra, and indeed all of mathematics is about generalizing patterns". Armstrong (1995) emphasized that exploring patterns at early gradesimprove children thinking abilities from an algebraic point of view, and signalized the importance of making generalizations in algebra using the patterns. Smith (2003), stressing on the relationship between patterns, functions and algebra, claimed that these three components must be integrated in the curriculum. In addition, many types of patterns – like linear and non-linear (quadratic and geometrical sequence), numerical, pictorial, arithmetical, geometrical and repetitive patterns – can be themes in the studies about pattern generalization including different education levels, from elementary schooling to pre-service school teachers, or from primary to secondary school (ORTON; ORTON, 1999; BISHOP, 2000; LANIN, 2005; RIVERA, 2007; AMIT; NERIA, 2008). In literature, great numbers of different types of patterns are studied using different strategies in order to generalize them. In order to determine such strategies, first of all, pattern types must be classified systematically. In this respect, Both Ley's and Feifie's studies about classifying different types of patterns are of huge importance. Ley (2005) puts forward linear patterns in five different formats as visual, geometric, table, number sequence and word problem. Feifie (2005), as seen in Figure 1, defines three different types of patterns and focuses on their five different formats.

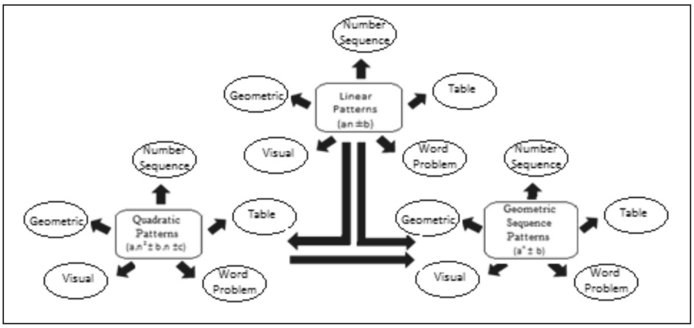


Figure 1 - Formats for different pattern types
Source: research data

The study of pattern generalization in school mathematics has been the focus of research conducted over the last years. In these studies, while focusing generalizing pattern types, it has been suggested that it should be focused on the detailed studies on strategies used in generalizing patterns. Stacey (1989), in his study about generalizing linear pattern problems with students in different educational levels, claims that they are more competent to find the near term than the far term in both shape and sequence of numbers. Orton and Orton (1999) defines patterns as a kind of approach leading to algebra and claims that students aged between 10-13 are much more able in generalizing linear patterns in sequences like “1, 4, 7, …” than generalizing non-linear ones, and they are also able to find near and far terms in linear patterns easily when compared with quadratic patterns. Feifei (2005), studying the ability of 8th grade students in problems including linear, quadratic and geometric sequences, finds out that these students are much more able to deal with linear pattern problems than with quadratic ones, and are also more able to deal with quadratic pattern problems than with geometric sequencespattern ones.

In addition to this, researches about generalizing different pattern problems – from primary school to university– put generalizing pattern problems in the center of the issue. In these studies, researchers define many strategies to deal with generalization in problems related to patterns such as recursive, common ratio, counting, additive, explicit or non-explicit, whole-subject, linear, guess-and-check, trial-and-error and contextual strategies, for instance (STACEY, 1989; ORTON; ORTON, 1999; BECKER; RIVERA, 2004, 2005; AMIT; NERI, 2008). Besides, all these studies, especially the one in which Stacey (1989) defines strategies in generalizing pattern problems,give foundation to many other studies.For instance,in a study about linear patterns, Stacey (1989) put forward some strategies about linear patterns including *near* and *far* generalization for 9-13 years students. The main contribution of her research was the classification of students’ strategies when solving contextualized linear generalization tasks, whether or not leading to correct answers. Strategies found were counting, whole-object, difference and linear. Based on this study, author concludes that a significant number of students used incorrectly a direct proportion method when trying to generalize. Another study based on Stacey’s work was held by Garcia-Cruz and Martion (1997). They developed a study to understand the processes of generalization of secondary school students. Their categorization of the methods used by these students was based on Stacey’s work. According to these researchers, there are three types of strategies: visual, numeric and mixed (numeric and visual). If the drawing had a fundamental role in finding the pattern, it would be assumed as a visual strategy. However, if the numerical sequence had a fundamental role for finding the pattern, then the strategy would be assumed as numeric. The students who used mixed strategies especially focused on the numerical sequence, but for confirming the validity of the solution they usually draw some sketch. This provides the setting for students that use visual strategies for checking the validity of the reasoning and for students that use numerical strategies. Moreover, Rivera and Becker (2005) claim that students mostly use numeric strategies, and describe three types of generalization: numerical, figural and pragmatic. Students that use numerical generalizationimplemented trial-and-error with little sense about what factors in the linear pattern mean. Students using figural generalization concentrated on relations between numbers in the sequence and could define variables under the scope of a functional relationship. Students that work based on a pragmatic generalization used numerical and figural strategies together in order to define sequences of numbers based on both properties and relationships. Ley (2005) describedthe three differentsolutionstrategies forlinearpattern problems: recursive, whole-object and explicit. According to Ley (2005), a recursive strategy involves the use of the previous term in the sequence to find the next term. Both, children and adults, commonly try to solve the problem finding the difference between two terms and then adding that result to the last term in order to determine the next number in the sequence. The whole-object strategy involves, often mistakenly, the proportional reasoning to solve question related to pattern. An explicit strategy involves generalizing the relationship between the two variables so that any value can be determined. This is the first step in a gradual progression towards expressing functions using formulas and equation. When using an explicit strategy, the rule is invariant and applicable to the near and the far terms, and therefore is conducive to create a general rule and reach the n^{th} term. Lanin (2005) analyses strategies of 6^{th} grade students that, in generalizing pattern problems, use a particular approach. Lanin's framework includes explicit and non-explicit strategies (non-explicit strategies are counting and recursive; while explicit strategies are whole-object, guessing and checking, contextual). In a more recent study, Amit and Neri (2008) focus on the strategies of middle school students (12-14 aged) in generalizing linear and non-linear pattern problems, and conclude that capable students have high mathematical abilities for pattern problems resembling generalization. They also declare that students use recursive method for local generalizationsand functional method for global generalization, in those cases that problems include both types of patterns. In addition, they state that students use, in general, additive and global strategies, and they emphasize that most of the students have preferred multiplication strategies to additive ones. Besides, most of the studies examining strategies of generalization of patterns study, generally, linear patterns. Only few of these studies examine non-linear patterns (KREBS, 2003; EBERSBACH; WILKENING, 2007; AMIT; NERIA, 2008). As an example, we can point out that according to the studies related to non-linear patterns (KREBS, 2003; EBERSBACH; WILKENING, 2007), additive strategies are mostly common and it has been stated that there has been a great tendency towards linearity, although the patterns are obviously non-linear. Moreover, while additive (expansive) strategies used in linear pattern problems are much more employed then a global generalization – because of the constant and obvious difference between each two successive patterns –, in non-linear patterns it can be said that this approach involving the use of visual approaches can prevent students from understanding the global structure (KREBS, 2003; RIVERA, 2007; AMIT; NERIA, 2008).

In many studies about patterns, it is observed that when students discover the relationship between the patterns and the generalization, it will help them to better perceive the world around them and, in addition, they can represent patterns in different forms and so they can make a major contribution on the transition from arithmetic to algebra, to algebraic thinking and it allows them to understand fundamental concepts of algebra. Thus, in the study carried out by the National Council of Teachers of Mathematics (NCTM's)Algebra Work Group, related to the future of algebra, it has been found out that algebra program in high schools is inadequate for the development of algebraic thinking, and this inadequacy results from the education in primary and secondary schools. They also have added that the sub-learning field of *pattern* is the key to solving this problem. So, according to Turkish primary mathematics education curriculum, studying students' efficiencies and strategies of generalization of different pattern types in primary schools are important for high school and university education in the future. In addition, as understood from the literature, the increasing studies including the abilities and strategies of the students from different learning levels on generalization of both linear and non-linear pattern problems will help put forward the *educational implementations* to be done for the transition from arithmetic to algebra.

In this context, the purpose of this study is to compare and determine 6^{th}-8^{th} graders' efficiencies, strategies and representations of student from different grades (from 6^{th} to 8^{th}) when dealing with problems related to linear and quadratic patterns.

2 Method

In this study, quantitative and qualitative methods were used together to collect and analyze data.

2.1 Participants

Our sample includes 246 students,from 10 to 15 years old, from an elementary school. Our data were collected in the second semester of 2008 – 2009 educational years in Trabzon, Turkey. 76 of those students are from 6^{th} grade, 74 from 7^{th} grade and 78 from 8^{th} grades. 18 of these students were chosen from different classes among the successful ones. Under the supervision of Turkish school administration, 18 students were selected to participate of clinical interviews. It was taken into account to select such students the representational model of the whole sample (students with similar and different classifications and levels of success in performing the tasks). When taking into account students' success levels in the initial phase of the study (student selection), students' results in Mathematics classroomwas given to us by their teachers. Thereafter, these grades have been classified into 3 groups. Furthermore, for the interviews, were chosen students who have the ability to express his/her thoughts fluently and who volunteered to participate.It is very important that the expressions of students to be clear and understandable for evaluation of the generalization strategies. For this reason, volunteer students who presented clear and understandable expressions are preferred.This can be seen in Table 1.

Table 1 - The sample of the research

		Test		Clinical Interview			General total
			Total	Weak	Medium	High	
Grades	Grade 6	76	228	2 ($S1_6$, $S2_6$)	2 ($S3_6$, $S4_6$)	2 ($S5_6$, $S6_6$)	246
	Grade 7	74		2 ($S7_7$, $S8_7$)	2 ($S9_7$, $S10_7$)	2 ($S11_7$, $S12_7$)	
	Grade 8	78		2 ($S13_8$, $S14_8$)	2 ($S15_8$, $S16_8$)	2 ($S17_8$, $S18_8$)	

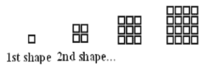
The abbreviations: ($S1_6 – S18_8$) are the codes of the interviewed students. (S_{ij}: i- Student Number; j- Grade)

2.2 Instrument

In the first step of preparing data, the relevant standards – especially

NCTM ones, accepted throughout the world– and literature related to pattern problem types were studied carefully. Thereafter, the classes in which the study would be carried out were determined taking into consideration the specific literature. Although there are many different types of patterns, mostly the sequence of numbers and shape formats of linear and non-linear patterns have been used in Turkish schools from 6th to 8th grades. In this context, the examples including different pattern types in TME can be seen in Table 2.

Table 2 - The examples including different pattern types in TME

Different Pattern Types	Number Sequence		Shape (Geometric+Visual)	
Linear Patterns ($a.n \pm b$)	3, 7, 11, 15, …	($4.n - 1$)		… ($3.n + 1$)
Quadratic Patterns ($a.n^2 \pm b.n \pm c$)	2, 6, 12, 20, …	($n^2 + n$)		… (n^2)

The help of an expert was called to determine whether the final shape of problems used to obtain data was suitable for the purpose of the measuring and whether it represents the field which was expected to be measured. Therefore, the problems prepared by a group of experts by analyzing the aims of the measurement and the content were discussed, trying to decide if they represent these aims and content. So, the problems have been examined by three Mathematics teachers and two Mathematics educators and the final corrections have been done in accordance to their suggestions. The language, level and extent analysis of the problems prepared according to the literature allowed us to use our instruments. Reliability is related to finding same or similar results when the research is being carried out in different periods or by different people. It includes examining the studied subject by more than one researcher in the course of the research or at the end of the research. For the reliability of data gathering means, students' papers have been taken by random and two researchers have coded them by taking into consideration the levels and strategies. As a result of the codification, 82% was the percentage of compliance among the two researchers involved. In addition, these are the problems which many students from different classes may bring up solutions, which many different strategies and multiple approaches may be used or even new strategies may be developed, and which one or more than one multi-thinking abilities can be included (STACEY, 1989; ENGLISH; WARREN, 1998; ORTON; ORTON 1999; LEY, 2005; FEIFEI, 2005). While the first two of these problems are linear, the others are non-linear. Besides, each problem was divided infive parts, calling the students to find the 1th, 10th and 40th terms; to expressthe rule of the pattern (orally or written); to write a rule or a letter for the term "n".

2.3 Data Analysis

While defining the students' efficiencies in being able to generalize patterns, the stages and the levels used in literature have been used. Three research studies of Orton, Orton and Roper (1999) and Orton & Orton (1999) were used as a guide to the researchers for data analysis when determining student levels about pattern problems. In this study, the researchers generated the levels presented in Table 3:

Table 3 - Generalization efficiency levels

Levels	Criteria
Level 0(L_0)	Nothing done in order to find a pattern
Level 1(L_1)	Find the first term in the pattern
Level 2(L_2)	Find the first and the 10th term
Level 3(L_3)	Find the first, the 10th and the 40th terms
Level 4(L_4)	Find the first, the 10th and the 40thterms; and express (written or orally) he rule of the pattern
Level 5(L_5)	Find the first, the 10 th and the 40 th terms; express (written or orally) he rule of the pattern and express the rule in an algebraic form.

Clinical interviews were done with students to determine what strategies they use in different pattern problems. Clinical interviews were done mutually, with students to understand their thoughts deeply. The main goal of this type of interview is to determine students' cognitive skills and find out the richness in their thoughts by revealing the students' concepts and the relation among these concepts. In these interviews, students are expected to complete the given goals on activity cards. Students' strategies and thoughts were defined in order to get answers for each goal and express – orally or in a written text – how they reach them (thinking aloud protocol), and to answer extra questions (How did you do it? How did you think? Why?). The problems have been presented to students on activity cards and during the interviews students' answers and the activities were tape-recorded. Interviews have been done individually in the last 80 minutes. Some direct quotations from the dialogues between the interviewer and the interviewed have been presented here, in terms of enabling the reader to make their own comments and see a descriptive and a more realistic picture of what and how interviews happened. The data gathered is classified and discussed by taking into consideration pattern generalization strategies in previous researches

(ORTON; ORTON, 1999; KREBS, 2003; LANNIN, 2005; RIVERA; BECKER, 2005; EBERSBACH; WILKENING, 2007; AMIT; NERI, 2008). These strategies are summarized in Table 4.

Table 4 - The frame of generalization strategies

Strategies	Contents
Counting	In order to count the number of pieces in a shape or measure the quality desired, a model or a shape should be pictured.
Recursive or Additive	It includes using the previous term in a pattern to find the next term or terms. Students generally try to find the difference between the two terms and add this difference to the last term to find the next term. Owing to continuing as repetitive and additive, this process is called additive strategy(adding).
Difference	It includes multiplying the difference between the two consecutive (sequential) terms in the series. This occurs especially in generalization of linear relationship, and students are aware of the stable difference between the terms. They describe the n^{th} term as multiplying the difference and "n" itself. This approach is valid for a sequence (series) as 3, 6, 9 ….. (3n), but invalid for a sequence 7, 11, …(4n).
Whole-Object	This strategy includes using proportional reasoning in solving pattern problems. Lannin (2003) describes this strategy as "using a portion as a unit to construct a larger unit using multiples of the unit; for example, if 3 apples are 9TL, 9 apples are 24TL.
Guess and Check	It includes a kind of prediction rule regardless of the rule working. An algebraic rule is put forward representing (symbolizing) the state of the problem. Students never think of the validity of the rule during the process. The algebraic structure that the students generate usually includes numbers and process about the problems.
Contextual	It includes configuring a rule focusing on the information that provides the case. This rule is associated with calculation technique.
Explicit	This strategy includes generalizing the relationship between the two variables in order to determine any value. This is the first step of a gradual progress to determine the functions by using equations and formulas. When this strategy is used, it becomes stable and practicable for both far and near terms, and so it helps find term "n" and write down a general rule.

3 Findings

The findings of the study aiming to define and compare the generalization efficiencies and strategies of different types of patterns of 6^{th}, 7^{th} and 8^{th} grade students are presented, in this section, in two parts with regards to the aims.

3.1 The generalization efficienciesof patterns of 6^{th}, 7^{th} and 8^{th} grade students

The efficiencies of the students related to the different types of patterns and presented above according to classes,were studied together, below. The column graphics, including the percentages belonging to the levels of 6^{th}, 7^{th} and 8^{th} grade students in test questions,are presented in Figure 2.

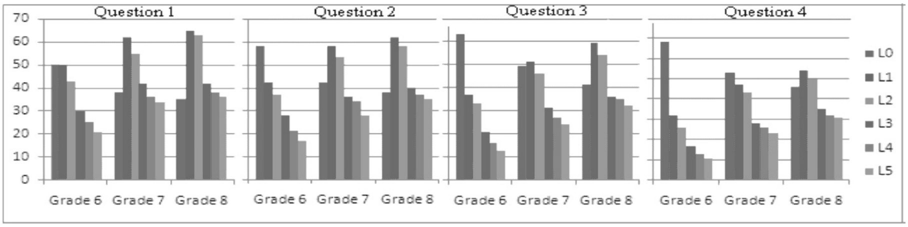


Figure 2 - The percentages belonging to the levels of 6th-8th grade students
Source: research data

It is seen in Figure 2 that when the learning degrees of the students increase, also the efficiencies related with the levels of students in every kind of questions increase. This positive increase can depend on the cognitive development and mathematical experiences of the students at different levels. In all levels related with the whole questions including different types of patterns, the percentage variance between 6th year students and 7th year students is more than the percentage variance between 7th year and 8th grade students. While the percentages of the 6th, 7th and 8th grade students expressing the pattern rule with letters in the first question related to linear number pattern are 21%, 34%, 36%, the percentages of those who expressed the pattern rule with letters in the second question related with linear shape pattern are 17%, 28%, 35%. Similarly, the percentages of the 6th, 7th and 8th grade students expressing the pattern rule with letters in the third question related with quadratic number pattern are 13%, 24%, 32%, and the percentages of those who express the pattern rule with letters in the fourth question related with quadratic shape pattern are 11%, 23%, 31%. In other words, the students in different learning levels are more successful in questions related to different types of linear patterns than the ones related to quadratic patterns. This can be associated to the situation in which the students can follow easily different types of patterns that they are familiar with. In addition, it can be seen that in all kinds of questions at the first two levels, when compared with the others, the students are more successful when finding the near terms than when finding the far ones.

The statistics about the efficiencies of 6th, 7th and 8th grade students in generalizing patterns are presented in Table 5.

Table 5 - ANOVA Results

Types of Pattern	Source	Sum of Squares	Df	MeanSquare	F	Sig.
Linear Number Pattern	Between Groups	26.010	2	13.005	2.855	0.060
	Within Groups	1 024.986	225	4.555		
	Total	1 050.996	227			
Linear Shape Pattern	Between Groups	30.827	2	15.363	3.479	0.033
	Within Groups	993.743	225	4.417		
	Total	1 024.469	227			
Quadratic Number Pattern	Between Groups	36.424	2	18.212	4.478	0.012
	Within Groups	914.993	225	4.067		
	Total	951.417	227			
Quadratic Shape Pattern	Between Groups	42.379	2	21.190	5.136	0.007
	Within Groups	928.371	225	4.126		
	Total	970.750	227			

According to ANOVA, statistically there isn't a significant difference between levels of the linear number pattern of the 6th, 7thand 8thgrade students. But there is a significant difference between the levels of linear shape pattern of the 6th, 7th and 8th grade students, considering the levels related to linear shape pattern, and the levels related to quadratic number pattern. The TUKEY test, defining the differences among these groups, is presented in Table 6.

Table 6 - The TUKEY test results

Dependent Variable	Group (I)	Group (J)	MeanDifference (I-J)	Standard Error	Sig.
Linear Shape Pattern	Grade 6	Grade 7	-.647	.343	.145
		Grade 8	-.860(*)	.339	.031
	Grade 7	Grade 6	.647	.343	.145
		Grade 8	-.213	.341	.807
	Grade 8	Grade 6	.860(*)	.339	.031
		Grade 7	.213	.341	.807
Quadratic Shape Pattern	Grade 6	Grade 7	-.623	.329	.143
		Grade 8	-.960(*)	.325	.010
	Grade 7	Grade 6	.623	.329	.143
		Grade 8	-.337	.327	.559
	Grade 8	Grade 6	.960(*)	.325	.010
		Grade 7	.337	.327	.559
Quadratic Number Pattern	Grade 6	Grade 7	-.600	.332	.169
		Grade 8	-1.046(*)	.327	.005
	Grade 7	Grade 6	.600	.332	.169
		Grade 8	-.446	.330	.367
	Grade 8	Grade 6	1.046(*)	.327	.005
		Grade 7	.446	.330	.367

(*) the average differenceis significant at 0.05 meaning level.

The ANOVA results,for repeated measurements, showing if there is a significant difference between the levels of students in different types of patterns, are presented in Table 7.

Table 7 - One-Way factorial analysis of variance results

	Source	Sum of Squares	Df	Mean Square	F	Sig.	Significant Difference
Grade 6	Between Subjects(A)	300.039	75	4.001			Linear Number Pattern and Quadratic Shape Pattern
	Between Measures(B)	18.382	3	6.127	1.76	0.016	
	Error (A x B)	780.618	225	3.469			
	Total	1 099.039	303				
Grade 7	Between Subjects (A)	352.662	73	4.831			
	Between Measures(B)	17.689	3	5.896	1.343	0.261	
	Error (A x B)	961.311	219	4.390			
	Total	1 331.662	295				
Grade 8	Between Subjects (A)	394.449	77	5.123			
	Between Measures(B)	8.987	3	2.996	.645	.587	
	Error (A x B)	1 073.013	231	4.645			
	Total	1 476.449	311				
General	Between Subjects (A)	709.893	149	4.764			Linear Number Pattern and Quadratic Shape Pattern
	Between Measures(B)	36.013	3	12.004	3.080	.027	
	Error (A x B)	1 741.987	447	3.897			
	Total	2 487.893	599				

According to the table, there is a significant difference between linear number patterns and quadratic shape pattern levels of 6thgrade students ($F_{(3\text{-}225)}$=1,76, p<0,05). It is impossible to confirmstatistically a significant difference between the answers of the 7th and 8th grade students for different types of patterns. Regarding to the 6th grade students, there is a statistically significant difference between the levels of linear number patterns and quadratic shape patterns of students ($F_{(3\text{-}447)}$=3,08, p<0,05).

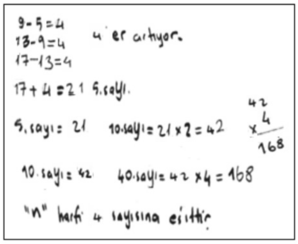
3.2 The generalization strategies of patterns of 6th, 7th and 8th grade students

In this part of the study, data gathered from clinical interviews are present according to different types of patterns.

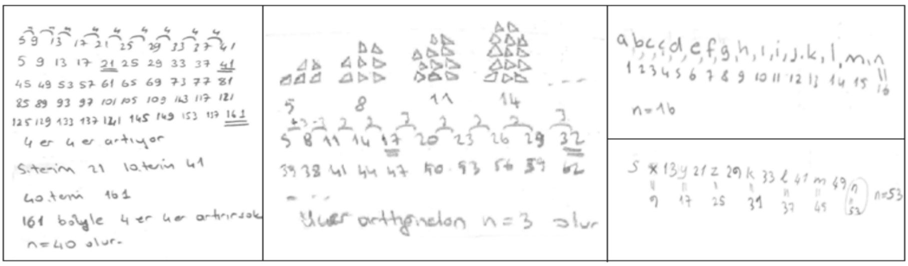
3.2.1 Generalization strategies of linear number sequence and shape pattern

$S1_6$, $S8_7$ and $S14_8$ students were able to turn linear shape pattern problem into number sequence pattern problem, determine the difference between two data,and accurately find the 5th term using difference. However, students found wrong values for the 10th and 40th terms due to the difference by using 5th term and "*whole-object (proportion)"*strategy. When the students were asked about the rule of the pattern, they stated that they only perceived the difference between two terms as a rule and that was unnecessary to written it because catching the difference among numbers was enough to find all terms. These students were able to make local generalizations and rise until L_1 level. In addition, the turning of shape pattern problem into number sequence problem indicates that these students only preferred numerical (arithmetic symbol) representations.

$S1_6$: In that case, if the 4th number is 17, the 5th number is 21. If the 5th number is 21,the 10th number is 42. [R: Why is 42?]… If the 5th number is 21, the 10th number is twice the fifth number because 10 is2 times 5. [R: What about 40?]… That is fourfold of 10th number. Multiply 42 by four. The result is 168. [A: How about the rule?]… There is a four-by-four increasing pattern. In other words, every number is four more than the other. [R: What is the n number?]… Teacher, the n number is four. [R: Why?]… We must find a number, and the numbers are increasing from four to four.

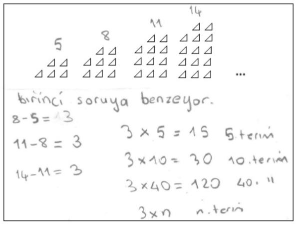


$S2_6$, $S3_6$, $S4_6$, $S7_7$ and $S13_8$ students were also able to turn shape pattern problem into number sequence problem, and they emphasized that the numbers in both problems were in a form that includes consecutive steps. However, they couldn't be successful in reaching a generalization and stating this as a symbol. Most of the students using the "*recursive (additive)*" strategy were able to find near terms accurately but unable to find far terms. As it has been seen on the solutions table next side, $S2_6$ and $S3_6$ tried to find the n-term of the pattern using the positional values of the letters in the alphabet. $S4_6$ and $S13_8$ students advanced to L_3 level, and $S2_6$, $S3_6$ and $S7_7$ students to L_2 level. In addition, all students used numerical (arithmetic) representations in the process of generalization of pattern problems.



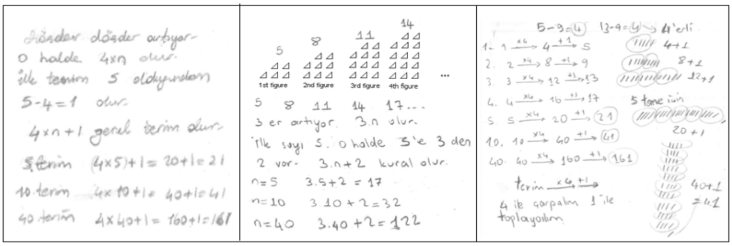
$S5_6$ student was able to find the differences between the terms of the patterns both in number sequence problem and shape pattern problem, and tried to find the solution by multiplying the acquired differences with the demanded terms. Trying to make generalization by multiplying the difference between two terms with the terms indicates that the student has used "*multiplying with difference*" strategy. Just like those students that used "*whole-object*" strategy, this student has reached the L_1 level and only used numerical representations.

$S5_6$: Firstly let's count the triangles in the shapes. 5, 8, 11, 14,… There are 3 differences among numbers all the time. In that case, the next numbers will be multiplied by 3. [R: Why, 3 times?]… to be a pattern. Anyway, if it isn't equal, we can't find the other numbers. In that case, the 5th number in the question is 15 (3 x 5 = 15), "the 10th number" is 30 (3 x 10 =30). [R: What about the 40th number?]… OK, then it will be 3 times 40, ant it is 120. [R: How can we write this as a rule?]…The pattern in which the difference is 3, or every number is three more than the other... So, the n number is "3 × n".



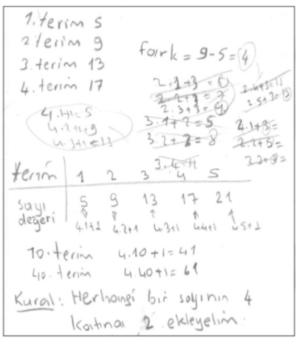
While $S16_8$ student was solving shape pattern problem, he stated that he learnt a formula used in solving these kinds of problems. The student pointed out that the formula would help him to answer the problem. $S16_8$ student, here, tried to solve the problem by using the formula which the student was familiar with, and included letter symbols instead of a known numerical value. In other words, the student used "*contextual strategy*". However, the knowledge of the student about the bounds of formula which were formed by the student when making binding between two variable terms and the place of the term is not at a cognitive level. Similarly, as $S16_8$, $S11_7$ also tried to make generalization for both problems by using a formula which was familiar to him. Even if the students tried to make generalization by using a memorized rule, they were successful in finding the correct generalizations. As it has been seen on the solutions table, $S9_7$ generalizes using the rule including 'the arrows' in the linear number sequence pattern problem. The student stated the generalizations using words, not letter symbols or algebraic expressions. $S16_8$ and $S11_7$ accessed the L_5 level and $S9_7$ the L_4 level. Herein, $S9_7$ used numerical (arithmetic symbol) and pictorial (line), and $S16_8$ and $S11_7$ used numerical and formal representations.

$S16_8$: A number sequence and an arithmetical sequence.[R:Why?]. The difference between the numbers remains stable. Let's use the formula including the n^{th}term instead of finding all terms one by one. [R:What kind of a formula?]... We have learnt it before. The formula appears from the correlation between terms and numbers… Since the difference is four, we should multiply 4 with n. [R: However, the 5th, 10th and 40th terms are being asked.]…If we use a general formula, values of all terms can be found. Multiplying 4 by n is 4 x n. Let's look at the first term. Since the first term is 5, we should look for the difference between four and five. [R: Why are you doing this?]… The formula… So, the number is 1. Now we should add 4 x n to 1. In that case the formula of this pattern is "4 xn + 1". Then, the 5th, 10th and 40th terms are 21, 41, 161.



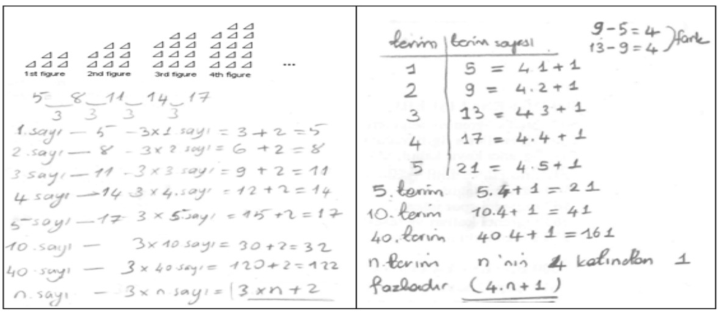
$S15_8$, using table representation, focused on the correlations between the term and the value of the term in the linear number sequence pattern problem. The student articulated the right rule by using "*guess and check*" strategy but he couldn't explain algebraically the rule. Similarly, as $S15_8$ student, $S10_7$ student solved linear number sequence pattern problem by using "*guess and check*" strategy. In the meantime, both of the students could form a number sequence pattern problem by focusing on the numbers of the triangles on the shapes of linear shape pattern problem, then they tried to make generalization by using "*guess and check*" strategy and the table, but they couldn't write the generalization as algebraic. Both of the students could access until L_4 level and they used numerical and pictorial representations.

$S15_8$: If I find the relationship between the terms and the numbers, I may find the numerical values of all terms. Let's draw a table, and then make the correspondence between the term in one side to the value in the other side. [R: What you want doing this?]… To make a well-ordered calculation. For instance, we should get 5 by using 4 and 1, which are differences between numbers. However, the rule we will find should be right for the others.. [R: How do you know ?]…This is the rule.



...[The student is making some post controls on the paper, here and after, using the results he thinks are correct result.]... In that case, 5 is four times the 1, and one more. It means that the rule is multiplying with four and adding 1. [R: OK, can you find n^{th} term?]. "n" is not a number, it is a letter, and we are finding here the numerical values of patterns. [R: How is that?]... For instance, if it is asked about the 100^{th} term, we must multiply 100 with 4 and add 1. The result is 401. In other words, we always find numerical values, and this rule can be stated by 'multiply any number with 4 and add 1'.

Besides, $S6_6$, $S12_7$, $S17_8$ and $S18_8$ had the knowledge on the formulas constituted by using the relationship between the place of the term and the two variable terms. $S17_8$ defined the general term of the pattern as "4.n+1" by using the table on the diagram next side and the relationship between two variables. All students pointed out the general terms algebraically, using "*explicit*" strategy. Similarly, $S12_7$ could turn linear shape pattern problem into number sequence pattern problem, and define the relationship between the term and the place of the term with the numbers by drawing, in a chart, one under the other, and write "3.n+2" for the general term. $S6_6$ and $S12_7$ used numerical and formal representations and $S17_8$ and $S18_8$ used numerical, pictorial (table) and formal representations. All of the students reached the highest level, L_5.

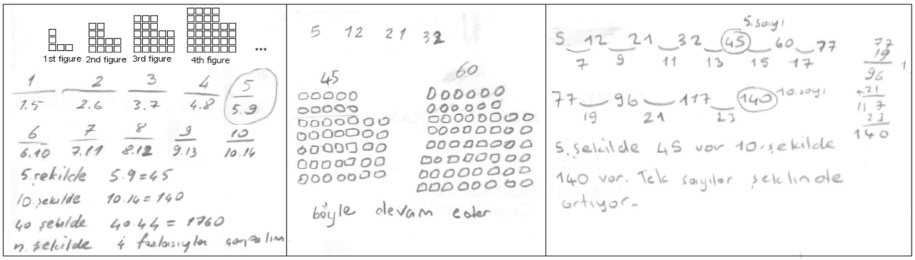


3.2.2 Generalization strategies of quadratic number sequence and shape pattern

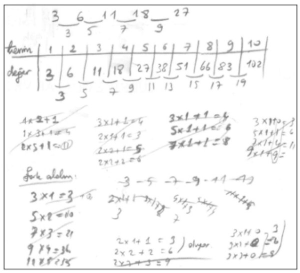
$S2_6$ could turn quadratic shape pattern problems into number sequence pattern problem and try to find out the result by using the difference between two terms in both problems. $S2_6$ indicated that there were only pattern types in which the differences were equal and he couldn't answer both problems. Turning shape pattern problem into number sequence pattern problem reveal that the student only preferred numerical representations. $S2_6$ is at L_0 level.

$S2_6$: Let's find the difference between the numbers, 3, 5, 7, … Is it a pattern, teacher? [R: Why it is not a pattern?]… The differences between the numbers are not equal. So, it isn't a pattern…Nevertheless, I will make some calculations. However, I can't find the result. [R: OK, let's focus on the other question.] Let's count the squares firstly, 5, 12, 21,.. This is not a pattern too, because the differences are not equal. [R: Does it always increase in the same way, using the same number?]… Yes, teacher. [R: Why?]… If the difference is not equal, we can't find the other numbers. Let's draw a shape and count.

$S3_6$ tried to adapt "*recursive*" strategy which includes numerical representations and focuses on the difference between two terms in quadratic number sequence pattern problem and only made local generalizations. The same student used "*counting*" strategy including pictorial(geometric) representations in quadratic shape pattern problem, and found the following first term by writing down the numbers of the squares in each pattern. Similarly, $S7_7$and$S14_8$, like$S3_6$, used "*recursive and counting*" strategies. $S3_6$, $S7_7$ and $S14_8$ rose to the level L_2dealing with quadratic number sequence pattern problem and to the level L_1 solving quadratic shape pattern problem. These students used numerical representations in number sequence pattern problems and pictorial(geometric) representations in shape pattern problems. $S4_6$ student could turn shape pattern into number sequence pattern. On the other hand, $S5_6$, $S6_6$, $S8_7$, $S9_7$, $S13_8$ and $S15_8$performed the same activity in contradistinction to$S3_6$, $S7_7$ and $S14_8$. $S4_6$ and the others students stated that the numbers of the pattern were mostly in the shape of consecutive odd numbers. These students used "*recursive*" strategy including numerical representations as they did in quadratic number sequence pattern problem and went to L_2 level. However, in quadratic shape pattern problem, $S5_6$ tried to find the result by using the difference between the term and the value of the term, and could find the 40^{th} term correctly, using the difference. So, he accessed the L_3 level. The students generally used numerical representations while solving both problems.



S10_7 focusedon the relations between the term and the value of the term by using the table representation in both of the quadratic pattern problems, but despite many test and check, he couldn't state the rule, orally or algebraically. The student used the table as a help to solve the problem, but he could find the terms until the 10^{th}, so he could reach the L_2 level. The student could write the solution on activity card by making the explanations for quadratic number sequence pattern problem. This case indicates that the student had the tendency to use "*guess and check*" strategy. He preferred numerical and pictorial (table) representations when solving both problems.

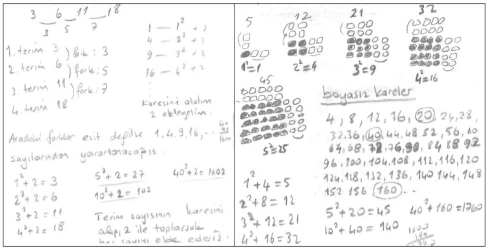


S10_7: The terms of the pattern are 3, 6, 11, 18 … If we find the relationship between terms and values, we can solve the problem. Firstly, let's draw a table putting the term on one side and the value on the other side. When the term is 1, the value is 3. When it is 2, the value is 6. When it is 3, the value is 11, and when it is 4, the value is 18. [R: Why did you draw a table?]… In order to better see the rule in the problem… to get a rule by finding the difference between the numbers. The rule should prove all of them. Let's find the differences. The results are 3, 5, 7 and 9,…But the difference is not the same. [R: What can we do?] Never mind, let's try to find the rule again. *[Although the student had performed many tests and checkings, he couldn't find the rule.]…*

While S11_7 and S16_8 were solving quadratic number sequence and shape pattern problems, they stated that they learnt a formula used in the solutions of these kinds of problems and defined that they could solve the problems by the help of this formula. S11_7 turned shape pattern problem into number sequence pattern problem and used "*contextual*" strategy using including, in both cases, numerical calculation techniques to structure the rule.As it has been seen below,

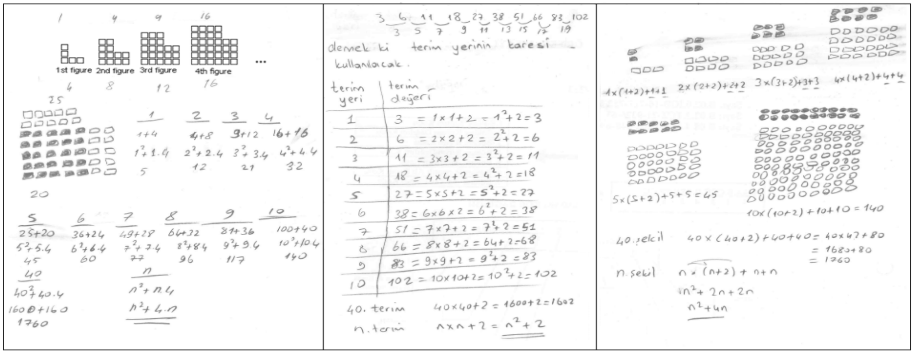
$S11_7$ stated the generalization using words instead of letters (maths) expressions. However, both the students, $S11_7$ and $S16_8$,were not capable, in a connectional level,to work with the formula related to the association between two variable terms and the place of the terms. These students could access the L_4 level when solving both problems and generally used numerical and formal (rule and formula) representations.

$S11_7$:The differences are not as it was in 3, 5, 7,…Since the difference is not equal, we will use the other formula. [R: Are there different formulas?] …Yes. If the differences are equal, there is a different formula. If the differences are not equal, there is another formula. In return, it is going on and on: the square of 1 is 1, the square of 2 is 4, the square of 3 is 9 and the square of 4 is 16… [R: What for these numbers are useful?] We will find the number which will be the difference between these numbers and 3, 6, 11, 18 … In that case, if we add 2 to the numbers such as 1, 4, 9, 16, the result will be 3, 6, 11, 18 ,... The fifth term is 27, the tenth term is 102, the fortieth term is 1602. [R: What is the rule?]… It is the square of the term plus 2.



$S12_7$ tried to form a rule by using the relationship between two variable terms and the value of the term. This student stated the relationships between two variables in the quadratic number sequence pattern problem with the help of a table and using "*explicit*" strategy, so he could define the general term of pattern as "n^2+2". $S17_8$ and $S18_8$ students also solved quadratic shape pattern problem by using both, numerical and pictorial, representations, and finding the relationship between the term and the place of the term. As $S12_7$, these studentsstated the general term as "$n^2 + 4.n$" using a "*explicit*" strategy. These three students are in a conceptual level of knowledge if we consider the use they did of the relationship between the term and the place of the term. They reach the L_5level in both of the problems, and they used numerical, pictorialand formal representations.

S12$_7$: Number sequence pattern. Firstly let's draw a table, so we will better see the relationshipbetween the place of the term and the value of the term. If we find the relationship, we can do the generalization as well. The square of the place of the term will be used since there are differences between the numbers. [R: Why?]… The process is done with the term itself if the differences are equal, and if the differences are not equal, the process is done with the square of the term. Under the circumstances, we will use the numbers continuing as 1x1, 2 x 2, 3 x 3, 4 x 4,,… OK. Let's think how we find 3 as the value of the 1st term. OK. If we take 1 multiplied by 1,it is 1, and we add 2 to that number, so the result is 3. Similarly, if we multiply 2 by 2,and add 2 to that number, the result is 6, but it is wrong. It means that the number which will be added is stable. Two. The number 2, has no relationship with the place of the term. In that case the first two multiplicands are changing according to the place of the term, and the addend number 2 is stable. OK. We find the rule. It must be to take the square of the term and add 2 to it. So, the 40thterm is 1760….Then, the rule is 'n multiply with n' plus 2. As a result, the rule is the square of n plus 2: n^2+2.



All of the data, including the findings gathered at this part, has been summarized at the Table 8 below.

Table 8 - All of the data including the findings gathered at this part

Students	Linear Pattern Problems						Quadratic Pattern Problems					
	Number Sequence			Shape (Geometric- Figural)			Number Sequence			Shape (Geometric-Figural)		
	Strategies	Levels	Representations	Strategies	Levels	Representations	Strategies	Levels	Representations	Strategies	Levels	Representations
$S1_6$	Whole-Object	L_1	Numerical	Whole-Object	L_1	Numerical	No Answer	-	-	No Answer	-	-
$S2_6$	Recursive (Additive)	L_2	Numerical	Recursive (Additive)	L_2	Numerical	Ineffective	L_0	Numerical	Ineffective	L_0	Numerical
$S3_6$	Recursive (Additive)	L_2	Numerical	Recursive (Additive)	L_2	Numerical	Recursive (Additive)	L_2	Numerical	Counting	L_1	Pictorial (geometric)
$S4_6$	Recursive (Additive)	L_3	Numerical	Recursive (Additive)	L_3	Numerical	Recursive (Additive)	L_2	Numerical	Recursive (Additive)	L_2	Numerical
$S5_6$	Multiplying with Difference	L_1	Numerical	Multiplying with Difference	L_1	Numerical	Recursive (Additive)	L_2	Numerical	Multiplying with Difference	L_3	Numerical
$S6_6$	Explicit	L_5	Numerical and formal	Explicit	L_5	Numerical and formal	Recursive (Additive)	L_2	Numerical	Recursive (Additive)	L_2	Numerical
$S7_7$	Recursive (Additive)	L_2	Numerical	Recursive (Additive)	L_2	Numerical	Recursive (Additive)	L_2	Numerical	Counting	L_1	Pictorial (geometric)
$S8_7$	Whole-Object	L_1	Numerical	Whole-Object	L_1	Numerical	Recursive (Additive)	L_2	Numerical	Recursive (Additive)	L_2	Numerical
$S9_7$	Contextual	L_4	Numerical and pictorial (line)	Contextual	L_4	Numerical and pictorial (line)	Recursive (Additive)	L_2	Numerical	Recursive (Additive)	L_2	Numerical
$S10_7$	Guess and Check	L_4	Numerical and pictorial (table)	Guess and Check	L_4	Numerical and pictorial (table)	Guess and Check	L_2	Numerical and pictorial (table)	Guess and Check	L_2	Numerical and pictorial (table)
$S11_7$	Contextual	L_5	Numerical and formal	Contextual	L_5	Numerical and formal	Contextual	L_4	Numerical and formal	Contextual	L_4	Numerical and formal
$S12_7$	Explicit	L_5	Numerical and formal	Explicit	L_5	Numerical and formal	Explicit	L_5	Numerical and pictorial (table)	Explicit	L_5	Numerical and pictorial (table)
$S13_8$	Recursive (Additive)	L_3	Numerical	Recursive (Additive)	L_3	Numerical	Recursive (Additive)	L_2	Numerical	Recursive (Additive)	L_2	Numerical
$S14_8$	Whole- Object	L_1	Numerical	Whole- Object	L_1	Numerical	Recursive (Additive)	L_2	Numerical	Counting	L_1	Pictorial (geometric)
$S15_8$	Guess and Check	L_4	Numerical and pictorial (table)	Guess and Check	L_4	Numerical and pictorial (table)	Recursive (Additive)	L_2	Numerical	Recursive (Additive)	L_2	Numerical
$S16_8$	Contextual	L_5	Numerical and formal	Contextual	L_5	Numerical and formal	Contextual	L_4	Numerical and formal	Contextual	L_4	Numerical, formal and pictorial (geometric)
$S17_8$	Explicit	L_5	Numerical, pictorial (table) and formal	Explicit	L_5	Numerical, pictorial (table) and formal	Explicit	L_5	Numerical, pictorial (table) and formal	Explicit	L_5	Numerical, pictorial (geometric) and formal
$S18_8$	Explicit	L_5	Numerical, pictorial (table) and formal	Explicit	L_5	Numerical, pictorial (table) and formal	Explicit	L_5	Numerical, pictorial (table) and formal	Explicit	L_5	Numerical, pictorial (geometric) and formal

4 Discussions and Conclusion

It was shown that when grade increases, students' efficiencies of generalizing pattern improve in a positive way in all levels. The improvement in that positive way might emanate from the cognitive development and the mathematical experience of the student. On the other hand, it has been confirmed that their efficiencies in generalizing pattern couldn't attain the expected level in any grade. The students in different learning levels are more capable dealing with linear patterns than of with quadratic ones, more able to deal with number patterns than with shape patterns. This result is nearly the same as the study of Orton and Orton (1999) and Feife (2005). The difference between our results and the results of these authors can be understood if we consider how teachers teach, generally focusing on some kind of problems that are similar to problems of linear number pattern. In other words, the students generally can get ahead easily in the types of pattern problems with which they are familiar. In addition, in general, students in all grades had more success in finding number patterns when compared to shape patterns. Besides, according to unilateral variance analysis (ANOVA), it has been found that there was no meaningful difference among 6th, 7th and 8th grade students in linear number sequence pattern problem, but statistically there were meaningful differences among these three classes' levels in linear shape, quadratic number sequence and quadratic shape pattern problems. According to the results of TUKEY test, these differences have been derived from the statistical differences among 6th and 8th grade students. Similarly, the findings acquired from frequency and percentage ranges also were used to support these results. In that case, as soon as the learning level increases, pattern generalization efficiencies of students from different learning levels change, progressing positively, although this change and progress are not satisfactory.

While the students from different learning levels were generally using *recursive (additive)* strategy for all problems (it includes finding the difference between two terms and adding the last term to the acquired difference in order to find the next term), *explicit* strategy was being used by few students. While five students used *recursive (additive)* strategy in linear pattern problems, ten students preferred to use *recursive* and *counting* strategies in pattern problems (it includes calculating the number of components of a shape or configuring a model depicting the case, in order to calculate the expected qualification or drawing a shape). Stacey (1989) has stated that most of the students put the focus on the difference only between two terms of the pattern in order to make generalization, and they get wrong generalizations. Three students from 6th grade, one student from 7^{th} grade and one student from 8^{th} grade used recursive strategy in linear pattern problems. On the other hand, five students from 6^{th} grade, three students from 7^{th} grade and three students from 8th grade have used *ineffective, recursive, counting* strategies in quadratic pattern problems. This data indicates that students preferred to use *recursive (additive)*strategies in which the difference between consecutive terms (that are convenient for linear pattern problems) was used even in non-linear pattern problem (quadratic and geometric sequence), and, in these cases, the students were unsuccessful. The students who used *recursive, counting* strategies could reach, at most, the L_3 level in linear pattern problems (only $S4_6$ and $S13_8$), and the L_2 level in quadratic pattern problems. Thus, test results collected from 228 students indicate that students generally made local generalizations (finding near terms, or reaching L_1, L_2, L_3 levels). When the column chart is examined (see Figure 2), we understand that the percentual difference (from 10% to 16%) among 6th and 7^{th} grade students (in all problems, including different pattern problems) is more than the percentualdifference (from 2% to 8%) among 7^{th} and 8^{th} grade students. ANOVA results indicates that, statistically, there was no meaningful difference among 6^{th}, 7^{th} and 8^{th} grade students only if we take into account the linear number sequence pattern problem, but there were meaningful differences among these three classes' levels in other problems. These differences were resulted from the statistical differences among 6^{th} and 8^{th} grade students. Such results could be tested and used as a guide during the interviews. According to the researchers, these types of strategies allow local generalizations in linear and quadratic patterns, but do not allow global generalizations (ORTON; ORTON, 1999; KREBS, 2003; RIVERA, 2007; AMIT; NERIA, 2008). In addition, all of the students using *recursive (additive)* strategy benefited from numerical representations, but when the students use counting strategy then can get better support from pictorial representations. This type of strategywas mostly used by the students in the weak levels. $S1_6$, $S8_7$, $S14_8$ and $S5_6$ students used *whole-object (direct proportion) and multiplying with difference* strategies in linear pattern problems, but since they didn't find a stable rate in quadratic pattern problems, they couldn't use *whole-object* strategy. On the other hand, $S5_6$ student used *multiplying with difference* strategy in quadratic shape pattern problem and reached the L_3 level. Due to the fact that these students only used these strategies since they were more convenient for some special pattern problems (like the sequence 3, 6, 9 …), they got wrong generalizations and could generally make only local generalizations. As a matter of fact these students in the weak and intermediate levels could reach the L_1 level in linear pattern problems and only used numerical representations. But $S14_8$ also benefited from pictorial (geometric) representations in quadratic shape pattern. Stacey (1989) and Lannin(2003, 2005) have stated that students using *whole-object* strategy were unsuccessful in generalizing pattern rule. In addition, these researchers have also remarked that the students understood the stable difference between terms, but, in order to make generalization, most of them used *multiplying with difference* strategy and, as a result, they found wrong generalizations. Our results are consistent with of these researchers. Besides, the students using *recursive, counting, whole-object and multiplying with difference* strategies had, generally, the tendency to convert shape pattern problems into number sequence problems. So, it seems that they want to use only numerical and pictorial (geometric) representations, apparently because they were negatively influenced while reaching global generalizations and because, in the process of the solution of complicated problems and especially shape and quadratic pattern problems, pictorial (figural) representations help them at the beginning of the problem and at the subsequent process, in order to organize the information (KREBS, 2003; RIVERA, 2007). $S10_7$ and $S15_8$ could reach L_4 level by using numerical and pictorial (table) representations, and *guess and check* strategy in linear pattern problems. $S15_8$ used numerical representations and *recursive* strategy in quadratic pattern problem and $S10_7$ used *guess and check* strategy in quadratic pattern problem. Both could access L_2 level in quadratic pattern problems. Besides, $S6_6$, $S9_7$, $S11_7$, $S12_7$, $S16_8$, $S17_8$ and $S18_8$ focused on the correlations between the term and the place of the term in all problems and used *contextual and explicit* strategies. $S9_7$, $S11_7$ and $S16_8$ used *contextual* strategy. These students tried to solve the problems using a formula which includes letter symbols instead of a known numerical value, and which they memorized or used before. The knowledge about the limits of the formulas formed by these students is not at a conceptual level. In addition, most of the students didn't find out the accuracy of the formulas they formed. Researchers have stated that students,generally, do not have the tendency to check the accuracy of the rules that they found (STACEY, 1989). However, $S6_6$, $S12_7$, S178 and $S18_8$ used *explicit* strategy, and so the knowledge about the limits of the formulas formed by making correlations between the term and the place of the term can be seen as being in a conceptual level. Since $S6_6$ and $S9_7$ use *explicit and contextual* in linear pattern problems and since, this way, they couldn't set a correlation between the term and the place of the term in quadratic strategies problems, they applied a *recursive* strategy. These students could reach the L_5 level in linear pattern problems, and the L_2 level in quadratic pattern problems. This can be related to the fact that these students progress easily in pattern types which they were familiar with (ORTON; ORTON, 1999, LANNIN, 2005; FEIFE, 2005). In TME curriculum, mostly, linear pattern problems have been emphasized. Also, it has been seen on test results carried out on 228 students that 8th grade students reach the $L_4$and L_5 levelsmore frequently than the 6th and 7th grade students, and there was less difference between the percentage values between 7th and 8th grade students than between students of other schooling grades. Results from clinical interviews show parallelism with these results because the number of 7th and 8th grade students reaching the $L_4$and L_5 levels by using *contextualand explicit* strategies in all problems is almost the same, and such quantity is larger than the number of 6th grade students. In *explicit and contextual* strategies, students generally used numerical, pictorial (table, geometric, line) and formal representations. This indicates that it is important for students to use pictorial and formal (rule and formula) representation at the same time because, in doing so, they could get successful generalizations. While students generally tries to solve four pattern problems using the same strategies, $S8_7$, $S9_7$, $S14_8$, $S15_8$ changed their strategies as long as the problems changed. Especially, while the learning level changed, *contextual and explicit* strategies were preferred, instead of strategies such as *recursive, counting, whole-object, multiplying with difference, guess and check*. 6thgrade students mostly focused on the difference between the terms but, on the other hand, 7th and 8th grade students focused on the correlation between the term and the place of the term. As a result, as long as the learning level increased, both solution strategies in the processes of generalizing patterns changed, and students' strategy sophistication increased. However, this change and increase is not at a satisfactory level. Students in different learning levels were more successful in generalizing linear pattern problems than in generalizing quadratic pattern problems and also more successful in generalizing number sequence pattern problems than shape pattern problems. It can be pointed out that these two differences is understandable since teachers concentrated mostly on linear pattern and number sequence pattern problems. Many researchers have declared that students are more successful in pattern problems which are familiar to them (ORTON; ORTON, 1999; LANNIN, 2005; FEIFEI, 2005). In addition, since some students from 6th and 8th grades tried to find near and far terms by writing the numbers in sequence or finding the difference between the terms and adding the difference to the previous term, these students were more successful in finding near terms than far ones. Hereunder, students' inadequacy in finding the far terms in the study contradicts with the proposal of NCTM (2000) "elementary students should find the far terms of the patterns since they are the basis of algebraic thinking". This result shows parallelism with Stacey (1989), Orton and Orton, 1999; Zazkis and Liljedahl (2002) and Ley(2005)'s studies.

While most of 6^{th} and 7^{th} grade students have the tendency of express the rules of patterns verbally or with sentences, 8^{th} grade students mostly preferred to use symbols. It indicates that as long as learning level increases, also increases the number of the students who expresses the rule algebraically or using letters. English and Warren (1998) Breiteig and Grevholm (2006) pointed out that in generalization problems – seen as a transition from arithmetic to algebra –, students who were in low learning levels have the tendency to express the rule of the pattern verbally, presenting arithmetical characteristics more. Therefore, it can be pointed out that 7^{th} and 8^{th} grade students who preferred words as representation are at a higher level than the 6^{th} grade students who preferred to use verbal representations. Many researchers have emphasized that expressing the rule of the patterns can develop algebraic thinking abilities of elementary school students. (STACEY, 1989; ARMSTRONG, 1995; CARPENTER; LEVI, 2000). The results from our study do not correspond with such researchers' advice.

When solving pattern generalization problems only few 6^{th} grade students used pictorial (table, line, geometric) representations, but 7^{th} and 8^{th} grade students frequently used such representations. When solving pattern generalization problems, especially $S10_7$, $S12_7$, $S15_8$ and $S17_8$ used chart and table representations as an effective means in the process of organizing and generalizing problems. The students tried to find generalizations by writing down the place of the term in one of the lines of the table and the number of terms in the other, taking into consideration the relation between these two variables. There are few students who use this type of pictorial (table, line, geometric) representations but these students were more successful than the others. Thus, Swafford and Langrall (2000) have stated that the use of tables in problem solving can help students and enable them to build a general point of view.

References

AMIT, M.; NERIA, D. Rising to the challenge: Using generalization in pattern problems to unearth the algebraic skills of talented pre-algebra students. **ZDM Mathematics Education**, v.40, n.1,p.111-129, Jan.2008.

ARMSTRONG, B. E. Teaching patterns, relationships and multiplication as worthwhile mathematical tasks. **Teaching Children Mathematics**, Reston, v.1, n.7, p.446-450, Mar. 1995.

BECKER, J. R.; RIVERA, F.An investigation of beginning algebra students' ability to generalize linear patterns. In: HOINS, M. J.; FUGLESTAD, A. B. (Ed.).Conference of the International Group for the Psychology of Mathematics Education.28th, 2004, Norway. P**roceedings…** Bergen, Norway: Bergen University College, 2004.v.1, p.286.

BECKER, J.R.; RIVERA, F. Generalization strategies of beginning high school algebra students. In: CHICK, H.L.; VINCENT, J.L. (Eds.).2005, Melbourne. Conference of the International Group for the Psychology of Mathematics Education, 29th, 2005.**Proceedings…** Melbourne, Australia: University of Melbourne, 2005, v.4, p.121-128.

BISHOP, J. W. Linear geometric number patterns: Middle school students' strategies. **Mathematics Education Research Journal**, v.12, n.2, p.107-126, Sep.2000.

BREITEIG, T.; GREVHOLM, B. The transition from arithmetic to algebra: To reason, explain, argue, generalize and justify. In: NOVOTNA, I.J.; MORAOVÁ, H.; KRÁTKÁ, M.; STEHLIKOVÁ N. (Ed.).Conference of the International Group of Mathematics Education.30th, 2006,Prague.**Proceedings…**Prague: Charles University, 2006, v.2, p.225– 232.

EBERSBACH, M.; WILKENING, F. Children's intuitive mathematics: The development of knowledge about nonlinear growth. **Children Development**. v.78,n.1, p.296-308, Jan.-Feb. 2007.

ENGLISH, L.D.; WARREN, E. A. Introducing the variable through pattern exploration. **Mathematics Teacher**, v.91, n.2, p.166-170, Feb.1998.

FEIFEI, Y. **Diagnostic assessment of urban middle school student learning of pre-algebra patterns**.Doctoral Dissertation, Ohio State University, USA,2005.

GARCIA-CRUZ, J.A.; MARTINON, A. Actions and invariant schemata in linear generalizing problems. In: PEHKONEN E. (Ed.). Conference of the International Group for the Psychology of Mathematics Education.21th, 1997, Helsinki. **Proceedings…** Helsinki,University of Helsinki, 1997, v.2, p.289-296.

KENNEY, P.A; SILVER, E.A. Probing the foundations of algebra: Grade 4 pattern items in NAEP. **Teaching Children Mathematics**, v.3, n.6, p.268-274, Feb.1997.

KREBS, A.S. Middle grade students' algebraic understanding in a reform curriculum. **School Science and Mathematics**, v.103, n.5, p.233-243, May.2003.

LANNIN, J. K. Developing algebraic reasoning through generalization. **Mathematics Teaching in the Middle School**, v.8, n.7, p.342-348, Mar. 2003.

LANNIN, J.K. Generalization and justification: The challenge of introducing algebraic reasoning through patterning activities. **Mathematical Thinking and Learning**. v.7, n.3, p.231-258, Jul.2005.

LEE, L. An initiation into algebraic culture through generalization activities. In:BEDNARZ, N.; KIERAN, C.; LEE, L. (Ed.). **Approaches to algebra**: Perspectives for research and teaching.Kluwer: Academic Publishers, 1996, p.87-106.

LEY, A. F. **A cross- sectional investigation of elementary school students' ability to work with linear generalizing patterns:** The impact of format and age on accuracy and strategy choice. Master Dissertation, Toronto University, Toronto: Canada,2005.

NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS (NCTM).**Curriculum and evaluation standards for school mathematics.** Reston, VA: NCTM, 2000. Available at: <http://www.nctm.org/standards.htm>. Access in: 07.03. 2007.

ORTON, A.; ORTON, J. Pattern and the approach to algebra. In: ORTON, A. (Ed.).**Pattern in the teaching and learning of mathematics**. Cassell: London, 1999, p.104 - 120.

ORTON, J.; ORTON, A.; ROPER, T. Pictorial and practical contexts and the perception of pattern. In: ORTON, A. (Ed.). **Pattern in the teaching and learning of mathematics**. Cassell: London, 1999, p.121-136.

RIVERA, F.; BECKER, J.R. Figural and numerical modes of generalizing in algebra. **Mathematics Teaching in the Middle School**, v.11, n.4, p.198-203, Nov.2005.

RIVERA, F. Visualizing as a mathematical way of knowing: Understanding figural generalization. **Mathematics Teacher**, v.101, n.1, p.69-75, Aug. 2007.

SMITH, E. Stasis and change: Integrating patterns, functions, and algebra throughout the K-12 curriculum. In: KILPATRICK, J.; MARTIN, W.G.; SCHIFTER, D.(Ed.). **A research companion to principles and standards for school mathematics**. Reston, VA: NCTM, 2003, p. 136-150.

STACEY, K. Finding and using patterns in linear generalizing problems.**Educational Studies in Mathematics**, Dordrecht, v.20, n.2,p.147–164, May.1989.

SWAFFORD, J. O.; LANGRALL, C. W. Grade 6 students' pre-instructional use of equations to describe and represent problem situations. **Journal for Research in Mathematics Education**. v.31, n.1, p.89-112, Jan. 2000.

THOMPSON, A.G.; THOMPSON, P. W.A Cognitive perspective on the mathematical preparation of teachers: The case of algebra. In:LACAMPAGNE, C.B.; BLAIR, W.; KAPUT, J. (Ed.). **The algebra initiative colloquium**. Washington, DC: US Dep. of Education, 1995. p.95-116.

TURKISH MINISTRY OF EDUCATION (TME).**6th-8th grade mathematics curriculum**. Ankara: MEB, 2005, p. 95-96.

ZAZKIS, R.; LILJEDAHL, P. Generalization of patterns: The tension between algebraic thinking and algebraic notation. **Educational Studies in Mathematics**, Dordrecht, v.49, n.3, p.379-402, Mar.2002.

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