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Knowledge Types Used by Eighth Grade Gifted Students While Solving Problems

Os Tipos de Conhecimento que são Usados por Alunos Superdotados na Oitava Série Durante a Resolução de Problemas

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Abstract

This study aims to determine how primary school eighth grade (14 years old) gifted students use knowledge types while solving problems. In the context, the data were collected through clinical interviews conducted with three gifted students. The students' voice recordings during problem solving and the solutions they wrote on the paper formed the data of the study. We found out that gifted students use more algorithmic knowledge and less schema knowledge in the problems that they had to solve. It can be said that the reduced usage of schema knowledge is likely to be a result of the fact that the gifted students produce different solutions using the field knowledge instead of remembering the schemas of similar problems they have encountered before.

Keywords: Problem solving. Knowledge types.Gifted students.Clinical interview.

Resumo

Este estudo pretende avaliar de que modo alunos com altas habilidades/superdotação na oitava série da escola primária (14 anos) usam os tipos de conhecimento enquanto a resolução de problemas. Neste contexto, os dados foram coletados por meio de entrevistas clínicas com três alunos com altas habilidades. As gravações de voz dos alunos enquanto a resolução de problemas e as soluções que eles escreveram no papel formam os dados do estudo. Verificou-se que alunos com altas habilidades usam o conhecimento algorítmico mais do que o conhecimento de esquema enquanto a resolução de problemas. Podemos dizer que menor utilização do conhecimento de esquema deve-se provavelmente ao facto de que alunos com altas habilidades produzem diferentes soluções usando o conhecimento de área em vez de lembrar os esquemas de problemas semelhantes que eles encontraram anteriormente.

Palavras-chave: A resolução de problemas. Os tipos de conhecimento. Alunos com altas habilidades/superdotação. Entrevista clínica.

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1 Introductıon

In the rapidly changing conditions of the world, many problems and incidents that we encounter affect our personal and professional life remarkably. The main skill that we should have in these conditions is not to cope with these problems on the surface level but to determine the key events about these problems, to acquire the required information and to produce effective and successful solutions for the problems, which are specific to us (BROUDY, 1982). That is, a qualified educational program should train people who are able to "solve problems". This requirement reveals the importance of focusing on problem solving as a thinking process in our educational system. The importance given to the problem solving skills in the education of an individual would help this person to make right decisions shaping his whole life (AKSU, 1989).

Problem solving does not mean only accomplishing the simple tasks that an individual encounters in his daily life. Problem solving includes more than remembering the simple tasks or the implementation of well-learned procedures (LESTER, 1994). It has an important place particularly in mathematics. Many educators state that problem solving is quite important in reaching the educational objectives and it should be the main objective of mathematics teaching in all levels of education (CHARLES; LESTER, 1984). There is a widely accepted perception in mathematics teaching that learning to solve problems actually develops the individuals' reasoning and analytical thinking and deepens their critical thinking. Problem solving simplifies learning mathematical phenomena, concepts, principles, and skills by making connections among them and implementing mathematical thoughts (PEHKONEN, 1991). Therefore, mathematics teaching and problem solving are concepts that should be considered together in contemporary understanding and problem solving is one of the main focuses of mathematics teaching. It is a key for understanding all the other fields of mathematics.

1.1 The knowledge types used in the problem solving process

The solution of a problem depends on not only computational skills, but also domain-specific knowledge (LOW; OVER, 1989). Mayer (1982a) mentions that an individual should have four types of knowledge in problem solving. These knowledge types are semantic knowledge, schema knowledge, algorithmic knowledge, and strategic knowledge. These knowledge types should not be dissociated.

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The first step in solving a problem is to understand the problem. The *semantic knowledge* of students is an important factor in this step. The student can turn the information in the problem into mathematical statements using his semantic knowledge (KARATAŞ; GÜVEN, 2003). If the participants are not competent enough in this area, they will have difficulties in forming the mental presentations accurately (MAYER, 1992). MacGregor and Stacey (1993) say that being incognizant of what the used variables represent leads to the inaccurate descriptions of those variables. Assigning the "x" variable to the unknown in the problems and explaining the problem with these variables, while understanding semantic knowledge, affect the problem solving process (STACEY; MACGREGOR, 2000). Therefore, one needs semantic knowledge in order to determine the given information in general, to define the problems by drawing, to use symbols like "x" for the unknown, to determine what the found value stands for, and to determine the required information for the solution.

Another type of knowledge, schema knowledge, is to express the information structures in a problem with similar problems or schemas while solving that problem. It depends on the principles and concepts inferred from previous experiences (PHYE, 1990). These concepts can be examined based on certain principles or characteristics, which serve in organizing a group of objects or samples of each category (ESTES; WARD, 2002). Schema knowledge is managed by associating the similar problem solving processes (GENTNER, 1989). According to Mayer (1992), schema knowledge means solving different word-problem types with the same arithmetic operations. When a problem solver knows what kind of a problem he is to solve, it means that he is using schema knowledge. This knowledge helps him distinguish relevant from irrelevant information. Riley and Greeno (1988) state that schema knowledge is necessary to plan the ways of solution. Kinstch and Greeno (1985) also mention the existence of schema knowledge that presents characteristics and relations in general in addition to children's understanding and solving the word problems. In the studies of schema knowledge, the participants are usually asked to classify the different types of word problems according to their mathematical processes (SWANSON; CONNEY; BROCK, 1993). When the schema knowledge is considered, in general, a student uses schema knowledge when he determines what kind of problem it is, when he determines whether the problem text is enough for the solution, when he determines the relationship between two variables, and when he states that there is too much information in the problem.

After the semantic knowledge and schema knowledge, the student will attempt to solve the equation and understand the problem. *Algorithmic knowledge* is necessary to solve

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the equation. The operational knowledge includes the arithmetic algorithm (SİMON, 1980). Algorithm means the path to be followed in problem solving. Mayer (1982a), however, defines algorithm as an accurate method in managing some operations like adding numbers. Mayer (2003) argues, as well, that algorithmic knowledge is necessary to manage the implementation and planning. Simon (1980) also states that algorithmic knowledge is an important part of problem solving. In this context, algorithmic knowledge emerges when a student knows the operations to apply for the equations, when he manages the required operations in solving equations, when he expresses the required statements while solving the equations formed to solve the problems.

Another knowledge type in problem solving is *strategic knowledge,* which is required in planning and control of problem solving. Strategic knowledge describes how the students analyze the problem, how they find the related content information, how they solve the problem, and how they plan (DE JONG; FERGUSON-HESSLER, 1996). Gathering the unknown on the one side and the known on the other side in order to reach a solution is the most commonly used strategic knowledge. A complex equation is transformed into a simple one by means of strategic knowledge (KARATAS, 2002). When all these definitions are considered, strategic knowledge is required for the student to express how he will solve the problem, to determine how he will utilize the mathematical statements, to leave the unknown in the equation alone, and to check whether his findings are accurate or not.

Mayer (1982b) explains these four types of knowledge as seen in Table 1 on the following problem.

<u>Problem</u>: *A boat travels for 120 minutes on a stream at 5km per hour; and it travels the same way at the same speed against the stream in 3 hours. According to this, what is the speed of the boat on the water?*

Table 1- Four types of knowledge that an individual can utilize to solve this problem.

Semantic Knowledge	Schema Knowledge	Algorithmic Knowledge	Strategic Knowledge
The river flow only downwards; however, the boat moves both upwards and downwards, 120 minutes is equal to 2 hours.	An individual decides that this is a movement problem, Forming the equation of (the speed of the boat + the speed of the stream).(the time period for the downwards)=(the speed of the boat-the speed of the stream).(the time period for	If R: the speed of the boat 2.(R+5)=3.(R-5) He decides which operations he will apply and how he will manage it after forming the equation above.	He finds the solution gathering the unknown values on one side of the equation and the known on the other side.

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the upwards)

When the studies on knowledge types are examined, it can be seen that they focus on the importance of knowledge types and how they are used. Some of these studies are as follows:

Low and Over (1989) have analyze in their studies whether the information given in a problem text include the required and enough information on algebraic problems. For this purpose, they asked tenth grade students to solve some problems. In each problem, the required information was missing and unnecessary information was given; and the students were asked to determine the unnecessary and missing information in these problems. This way, the schema knowledge of the students was evaluated. They found out that 90% of the students could define the problem text accurately and an accurate definition of the problem text was the determinant factor in problem solving. Low and Over (1992), in another study, presented problems related to the area of the rectangle to 195 students enrolled at ninth and tenth grades; and they asked the students to determine whether the problem text was enough to solve the problem and to determine the unnecessary information in the text. This way, the schema knowledge of the students was evaluated; and it was found that the students who were unsuccessful in less difficult problems were not even aware of the more difficult ones.

Geiger and Galbraith (1998) have studied the importance of knowledge structure and the effects of the beliefs in the problem solving process in their studies. At the end of the study, they found that the students who could not reach the solution of the problem or could not form the equation to solve the problem were inadequate in terms of their knowledge types. However, it was determined that the students who could define the problem effectively and reach the solution in an organized way were much better in determining the knowledge types. Karatas (2002) analyzed the problem solving process qualitatively using a clinical interview method with five students enrolled in eighth grade; he attempted to find out how the students used the knowledge types in the problem solving process. The students effectively using the knowledge types in problem solving process were successful both in forming the equation for the problem and in finding the accurate solution. It was determined that strategic knowledge

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was an important knowledge type in the evaluation step. The students who made operational mistakes while solving equations could correct their mistakes and find the solution as they utilized the appropriate strategic knowledge. Karatas and Güven (2003) also studied how the eighth grade students used knowledge types in problem solving steps. In the study, six problems were asked to five students through clinical interview and it was found out that the students who used the problem solving steps successfully were the ones who were using the knowledge types effectively. As a result, it was stated that the use of knowledge types effectively was required to develope students' problem skills.

1.2 Giftedness and solving problem

Individuals in a society have different levels of intelligence and ability. Almost 5% of the population of a society is composed of gifted individuals and individuals with learning disabilities. Of this group of 5%, almost 2-3 % of them are highly talented and gifted people (MARYLAND, 1972). Various educators have defined the concept of giftedness differently and they attempt to explain it with different parameters. Gifted people are not a different type of people; they are individuals who are different from others in terms of distribution, frequency, timing, and composition of the characteristics, which exist in all human beings (AKARSU, 2001).

It is inevitable that gifted students have different abilities. According to some studies, in terms of mathematics, it has been found out that gifted students are successful in problem solving processes like organizing materials, using templates and rules, modifying the problem statement, using new expressions in templates and rules, understanding and studying on very complex issues, reversing the processes, and finding relevant problems (MILLER, 1990). Moreover, gifted students, in terms of mathematics, are the ones who exhibit mathematical skills which older students can do (SOWELL, et al., 1990). However, solving problems quickly and memorizing symbols, numbers and formulas cannot be regarded as an indicator of giftedness (WİECZERKOWSKİ; CROPLEY; PRADO, 2000).

As the quantitative and qualitative thinking skills of gifted students are more developed, their problem solving skills are much better than of those of ordinary students (KNEPPER; OBRZUT; COPELAND, 1983). Therefore, it has been observed that gifted students tend to be better mathematical problem solvers than the ordinary students of the same age (GALLAGHER, 1975; RENZULLI, 1978).

When we reviewed the studies on the problem solving processes of gifted students, it can be seen that, in order to identify a gifted student, these studies focus on how gifted students at secondary schools solve the mathematical problems, which are not routine and their way of posing a problem. Some of these studies are as follows:

Most researchers have studied how secondary school gifted students solve the mathematical problems which are not routine. They have found out that gifted students spent more time while they are rereading and interpreting the problems according to their own words (GAROFALO, 1993; SRIRAMAN, 2003). Düzakın (2004) has found in his studies that gifted students could make connections among the ideas, which seem irrelevant from each other, could conceptualize the abstract things in problem solving process, and have skills for synthesis. Keşan, Kaya and Güvercin (2010), in another study, aimed to give some directions for teachers to improve their students' level by using a problem posing approach, which has two dimensions. First special problem posing tasks were prepared for students and second, face-to-face interaction with them. As a result, the usefulness of this approach will be discussed for secondary school teachers in order to use this method in their special courses and, secondly, how a special curriculum can be prepared for gifted students. Finally, they have found out that the method of problem posing may be used in the identification process of a gifted student. Heinze (2005) has also found in his studies that mathematically gifted elementary students stand out in the ability to work systematically and quickly, getting an insight into the problem's mathematical structure. Additionally, these children stand out in their high ability to verbalise and to explain their solutions. In comparison to non-gifted children, these qualities show significance pertaining to problem solving.

1.3 The purpose of the study

The determination of the knowledge types used by gifted students while solving problems may be a guideline while determining the characteristics of these students. Thus, some points about the differences between these students and ordinary students may be determined and this may help the authorities during selection procedure. This study aims to find out how primary school eighth grade gifted students use knowledge types while solving mathematical problems. In this context, the problem of this research is "how do the gifted students in the eighth grade of primary school use the types of knowledge when solving mathematical problems?". The sub-problems are;

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1. How do the gifted students in the eighth grade of primary school use the semantic knowledge when solving mathematical problems?
2. How do the gifted students in the eighth grade of primary school use the schema knowledge when solving mathematical problems?
3. How do the gifted students in the eighth grade of primary school use the algoritmic knowledge when solving mathematical problems?
4. How do the gifted students in the eighth grade of primary school use the strategic knowledge when solving mathematical problems?

2 Research and design

As a certain group is examined in detail and the data is obtained through the data collection tools, it had no purpose for generalization, so a case study method was used in the study. The interviews were called clinical interviews in order to analyse the behaviors of the students deeply.

In Turkey, gifted students are educated at Science Art Centers (SAC), which are independent from formal school programs. The study has been carried out with three students (two of them are male, one of them is female) who were enrolled at Science Art Center. Of these participants, two of them had been going to the Science Art Center for 3 years beginning in sixth grade (12 years old) and the other one had been going to this center for 4 years beginning in fifth grade (11 years old). All of the participants were among the successful students at their schools and all their grades for the Fall Term were 5 out of 5.

2.1 Data collection

In the data collection of this study, a clinical interview was used. The case study researches an appropriate situation, whose limits are certainly determined (STAKE, 1976). The questions to be asked in the clinical interview were determined clearly. After that, it was paid attention to the fact that more than one problem solving strategy could be used in the problems asked to the students. Thus, an examination of knowledge types that students use while solving problems within a narrow framework was not limited with only one problem solving strategy. The problems were prepared using the mathematics curriculum and mathematic course books. These prepared problems were chosen from the problems, which

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took place in the courses after discussing with the mathematics teachers at the Science and Art Center. After the clinical interview, the questions and problems prepared by the researchers were reviewed by two field experts, and they were implemented after making the required modifications.

The students knew the purpose of the study superficially before starting the clinical interview. Each interview was completed in the guidance room, which was a silent environment that students felt comfortable in almost an hour's time. The purpose of the clinical interviews was to determine the knowledge types that students used while solving the given problems. Therefore, the students were asked some questions such as *"what is the most important information that could help you in solving the problem?", "is the problem text enough for leading to the solution?", "what are you planning to do before solving the problem?", "what have you found now?"* etc.

2.2 Data analysis

The gifted students were asked to solve three problems. The responses given to the questions were recorded during the interview. The data was documented and controlled before the analysis of the data obtained in the study. The responses of the students for the questions in the clinical interview were used in order to determine the knowledge types that students used in problem solving process. Therefore, the knowledge types used while analyzing the problems were given in Appendix-1 and the knowledge types students used while solving problems were determined as in the sample about the 1st problem and frequency tables were created in Appendix-2.

3 Results

In this section, we show the knowledge types that the three primary school eighth grade gifted students used in the problems which were asked to them. The findings were supported with the direct quotations taken from the clinical interviews and the solutions students wrote on the paper. These findings were interpreted for each student separately and it aimed to determine how gifted students used the knowledge types in problem solving. On the other hand, the findings were presented assigning pseudonyms to the participants.

3.1 The case of Ali

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Analysis of the knowledge types used for the problems by Ali is presented in Table 2.

Table 2 - The frequency of knowledge types Ali used while solving problem.

ALİ	Semantic Knowledge (f)	Schema Knowledge (f)	Strategic Knowledge (f)	Algorithmic Knowledge (f)
First Problem	*4*	*2*	*4*	*6*
Second Problem	*9*	*3*	*7*	*5*
Third Problem	*4*	*4*	*3*	*5*

Ali used less algorithmic knowledge than the other students did. This can be associated with the fact that Ali used the required information less in solving the equation that he formed for solving the problem.

Ali defined the problems in first and third questions by drawing as in Figure 1 and Figure 2, and he differed from other students in the way that he produced solutions. This could be caused by the fact that Ali made an effort to solve the problems using different ways.

(11) A: How can you solve it in other ways?

(12) Ali: I will draw a square; I will solve it using the area as it is a square.

(13) A: Let's see. What are you doing now? Are you drawing a square that is 20 cm long?

(14) Ali: Now, here the square of 20 is equal to its total area, however, the square of 19 is equal to the area I have drawn, thus, only this external area remains to me.

(A: Researcher)

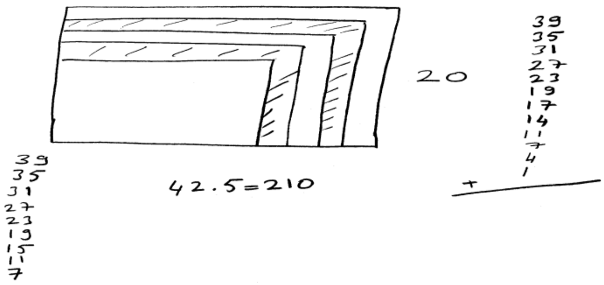

Figure 1 - Ali's definition of the first problem drawing figures and using semantic knowledge.

(19) A: Can you solve this in another way?

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(20) Ali: Yes, I can do it by drawing as well. Let this be 1st, 2nd, 3rd, 4th, 5th and 6th color. Let me put the same ones here as well. We will match among these but one cannot be with one and two cannot be with two…. Six cannot be with six. The rest of the matches are possible; however, half of them might be the same. One with two and two with one are the same and I will leave the same ones. This matching and that matching are the same so this half is the same with the other half so I will take half of them. 1, 2…15.

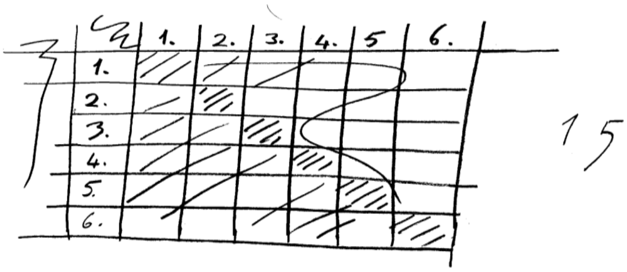

Figure 2 - Ali's attempt to find a different solution.

On the other hand, the strategic knowledge, as seen in Figure 3, that Ali used in the second problem is different from the other students as he gave up thinking that the way he wanted use to solve the problem would be wrong. This is important in terms of his making decisions whether the plan he was going to prepare would be an appropriate plan or not after he had understood the problem.

(15) A: Can you solve it in another way, think about it?

(16) Ali: Is it possible if I draw it? Let this be x height, firstly I started from this height. I divided into three equal parts. All in all, 1st triangle is here, it rises up to this point. This will be divided into two parts.

(17) A: Yes.

(18) Ali: Here, I divided into 3 equal parts again. Actually, I may not calculate as the length is something.

(19) A: You can try, we have time.

(20) Ali: I will try, then. This equal parts 1, 2, 3 will be divided into equal parts after the fall. If the rest parts are divided into 3 parts, I should find what fraction the fourth part is. However, I do not find the total length in order to do this. Or, I may not draw straight. I cannot reach the solution from here. I should think about a different method.

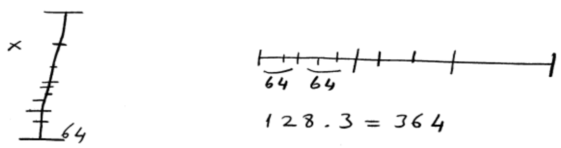

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Figure 3 - The mistake Ali did in the problem.

3.2 The case of Veli

The knowledge types used for each problem by Veli are given in Table 3.

Table 3- The frequency of knowledge types Veli used while solving problems.

VELİ	Semantic Knowledge (f)	Schema Knowledge (f)	Strategic Knowledge (f)	Algorithmic Knowledge (f)
First Problem	*1*	*3*	*4*	*14*
Second Problem	*8*	*2*	*3*	*8*
Third Problem	*5*	*2*	*3*	*7*

Veli was the student who used schema knowledge the least. Particularly, the second and the third problems were the problems that required the use of schema knowledge more. But schema knowledge was used only twice in the second and third problems as in Figure 4.

(3) A: So, what do you think about the question?

(4) Veli: It can be drawn with combination.

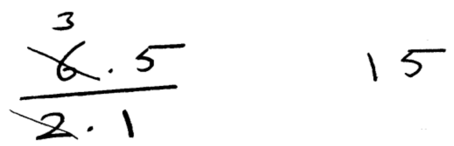

Figure 4 - Veli's making a decision on what kind of problem the third problem was and his reaching the solution.

The student who used the strategic knowledge less than the other students in the second problem was Veli. However, strategic knowledge should have been used more in the second problem; because, a student can easily express how to solve the problem in this problem, determine how he will use mathematical statements, leave the unknown alone, and check how the result is correct. The other students used strategic knowledge more than Veli did; and this was caused by the fact that they expressed how they would solve the problem explicitly.

Veli used semantic knowledge for once in the first problem. However, the students were expected to determine the given information and what the found value stand for in order to make them use semantic knowledge.

On the other hand, Veli used algorithmic knowledge in the problems very frequently as in Figure 5. As for the reason for this, it can be considered that the student had known the

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operations that he applied to the equations and, thereby, he did the operations correctly while solving the equations.

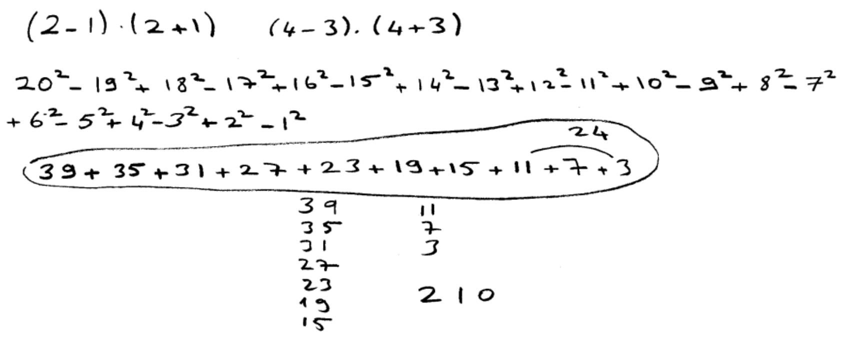

Figure 5 - The operations Veli did in order to solve the equation in the first problem.

3.3 The case of Ayşe

The knowledge types used for the problems by Ayşe is in Table 4.

Table 4- The frequency of knowledge types Ayşe used while solving problems.

AYŞE	Semantic Knowledge (f)	Schema Knowledge (f)	Strategic Knowledge (f)	Algorithmic Knowledge (f)
First Problem	*1*	*3*	*1*	*4*
Second Problem	*9*	*3*	*4*	*7*
Third Problem	*3*	*2*	*3*	*8*

As it can be observed in other students, Ayşe also used schema knowledge very little. The reason for using schema knowledge this little might be caused by the fact that Ayşe usually wanted to solve the problem directly.

(5) A: What do you think before solving this problem (third problem)?

(6) Ayşe: If I have six unknown numbers, I will choose two of them. It says there are two pairs. How much can I find according to this? I will think this as six colored circles. I will write six different things. These things will be matched 2 by 2. I will multiply the sum. From here, you do two like this; so this is necessary.

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Ayşe was the student who used the strategic knowledge less than the other students did. She used this knowledge less and this might be caused by the fact that she immediately started solving before stating how to do it.

On the other hand, while Ayşe used the semantic knowledge frequently in the second problem as in Figure 6, she used it less in other problems. This might be caused by the fact that in order to solve the second problem, the students naturally determine the given information, define the problem by drawing it and use symbols like x for the unknown numbers.

(3) A: Have a look at this… Just say what you have understood from the problem.

(4) Ayşe: Ball, it is… A height is given at the beginning and the ball rises 2/3 at this height, the height becomes 64 until the fourth time.

(5) A: Then, it bounces, doesn't it, it becomes less and less. Finally, it rises to 64 meters. Ok. What do you think is the most important information in this problem?

(6) Ayşe: I think xs. I would do… I would give a value at the beginning in this problem; that is, I would give a x close to these values and I would move on according to this x, how many xs it is equal to in the fourth one, 64 is equal to that one as well.

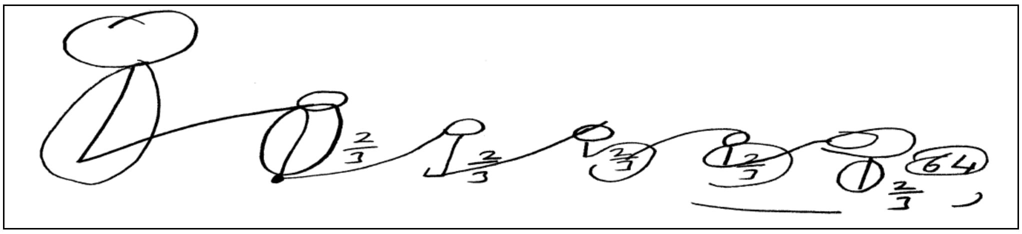

Figure 6 - Ayşe's defining the second problem by drawing.

When the gifted students' use of knowledge types in problem solving process is examined in general, it was observed that they used more algorithmic knowledge and less schema knowledge in the problems they were asked to solve. The reason for the use of more algorithmic knowledge might be caused by the fact that these students had a good command of mathematical operations and they could easily determine which operations and where they were used.

4 Conclusion and discussion

In this part of the study, with the purpose of illuminating how gifted students in the eighth grade of primary school use the types of knowledge while solving mathematical

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problems, the findings were supported with other studies in the literature and discussed in the context of sub-problems.

4.1 Discussion on the types of knowledge

When the knowledge types the gifted students used while solving the problems were examined, it was observed that algorithmic knowledge was more used and in a comfortable manner. Lester and Kroll (1990) stated that students could reach the solution when they interpreted the problem and turned it into mathematical equations. Therefore, it should be managed that students should firstly understand the problem accurately and correct their mistakes, if there are any, in their operations in order to solve the problem correctly.

Mayer (1982a) mentioned that the main difficulty students experienced about problems was in understanding the question. However, it was observed that the gifted students were good at restating the problem with their own words using their semantic knowledge in problem solving. The findings that students could express themselves better by drawing and reaching the solution more easily is parallel with the finding of Hong (1993) that students could express the problem better by forming a model. A student who would like solve the problem correctly should try to understand it, try different ways for this purpose and after that he should move to the solution.

Gifted students are characterized by their ability to give sustained attention to problem solving; by their propensity to question, experiment, and explore; by their inventive solution strategies (JOHNSEN, 2004). It was also seen that the gifted students in this research could correct themselves when they found the wrong answer. This could be explained with the fact that the gifted students could check the correctness of their answer using strategic knowledge. Also, Steiner (2006) suggests that gifted students know more and better strategies, are more flexible in their use, and are more likely to choose effective strategies. It is natural that this finding is different from Taconis (1995) and Mathan and Koedinger’s (2005) finding that regular students had difficulties in strategic knowledge while solving problems. This is a point that should be thought on for many educators who thought that students could not solve the problems because of the lack of knowledge. Therefore, educators should help their students about how they could use their knowledge and how they could plan for the difficulties, they encountered while solving problems.

When the knowledge types the gifted students used while solving the problems were examined, it was observed that they used the schema knowledge the least among the four

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types of knowledge. Low and Ower (1989) stated in their study that schema knowledge, which was about the determination of the information for solving the problem should be used and they mentioned that the students using this knowledge were successful. Kroll and Miller (1993) and Mayer (1982a) found out that regular students used the structure of the problem they solved before in order to solve the problem. However, the gifted students became successful although they used the schema knowledge very little. In the light of this finding, it can be claimed that the gifted students focus on developing different ways of solution instead of associating the problem with similar problems; because a student who could develop different ways of solution has the opportunity to have a look at the problem from different perspectives. Therefore, students should be educated to think of different ways of solution during the problem solving process.

4.2 The types of knowledge used by gifted students

Sheffield (2003) argued for the use of open-ended mathematical tasks that aim to make gifted students think. The students who participated in this study succeeded in thinking processes while trying to solve word problems. Gifted students are idiosyncratically well-equipped and this may have played a role in their success in the face of such conditions that normally complicate the thinking processes.

Mathematical activity that helps students experience leaps in understanding, or surprise, or that supports students to use their imagination is not typical in mathematics education (GADANIDIS; HUGHES; CORDY, 2011). Therefore, as this study suggests, teachers should help their students to organize the thinking processes while solving problems. In his PhD thesis, Yıldız (2013) studied on improving the behaviours of teachers in activating their students’ metacognition in problem-solving environments. In this context, he carried out lesson studies with teachers.

When the factors affecting problem solving process were examined, it was found that the problem solving achievement was not only affected by the computational skills but also the knowledge that the individual had (LOW; OWER, 1989). In this study, which was also carried out with the gifted students, it was seen that the students who could use knowledge types in an effective way were successful in problem solving. What the students who could use knowledge types effectively mean here is that analyzing the problem, remembering the previously solved problems, preparing plans for solving the problem, conducting appropriate operations for the plan, and reviewing what is done at the end of this process. Therefore,

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students should be trained as good problem solvers giving importance to better use of knowledge types besides the problem solving steps and strategies in problem solving process.

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Appendix-1.

The analysis of knowledge types that should be used for each problem.

	Semantic Knowledge	Schema Knowledge	Algorithmic Knowledge	Strategic Knowledge
First Problem: $20^2-19^2+18^2-17^2+........+2^2-1^2=?$ What is the solution of this operation?	Trying to solve the problem by drawing squares Stating that the found 210 value is the addition of two square differences	Stating that the problem is a difference of two square problem Expressing that the problem text is enough for the solution	Finding a 210 value using the difference of two squares Finding a 210 value using the areas of the squares	Stating that the problem can be solved using the difference of two squares Stating that the problem can be solved by drawing squares Checking the result
Second Problem: A ball thrown from a certain height rises 2/3 of the previous height when it hits the ground. If the ball rises to 64 meters after it hits the ground four times, from how height is the ball thrown?	The determination of the information that will help solve the problem is that the ball rises 2/3 of the previous height and the ball rises to 64 meters after it hits the ground for four times Trying to find the solution by drawing it Expressing that 324 meters is equal to the height at the beginning	Thinking about similar problems solved before Examining whether the given prompts are enough to find what is asked for	If x is the height from which the ball is thrown, finding the height of the ball after it hits the ground 4 times and finding x as 324 after solving the equation 16x/81 =64 Beginning from the fact that the height of the ball after the 4^{th} hit is 64 meters 64.(3/2).(3/2).(3/2).(3/2) Finding 324 meters after the operations	Expressing how he will solve the problem beginning from the fact that the ball rises to 64 meters after the 4^{th} hit After checking the 324 meters solution, trying to reach 64 meters Trying to find the solution giving value to the height at which the ball is thrown at first

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Third Problem: A building contractor gives a catalog to the house owner to choose the color of room parquet. There are six different colors for the parquet in this catalog. The house owner will choose two of these colors. According to this, how many pairs of colors can he choose?	Determining the information that will help to solve the problem is that 2 colors out of 6 colors can be chosen Trying to solve the problem drawing lines indicating the colors Stating that the result is equal to the color pairs that we can choose in different ways	Deciding that the problem is a combination problem	If n: 6 different color choices r:2 different selected colors $\frac{n!}{(n-r)!.r!} = \frac{6!}{(6-2)!.2!}$ $\frac{6!}{4!.2!} = 15$ Finding the solution Solving the problem in different ways without using the formulas	Stating that the problem can be solved with combination Checking the result

Appendix-2.

The Students' Interview Data and the Knowledge Types They Used for the 1st Problem

Student	The Statements of Gifted Students	
Ali	(1) A: Which is the most important information that chould help in problem solving? (2) Ali: The Square of 20 minus the square of 19. Here, we have the difference of two quares. (3) A: Why did you think so? (4) Ali: The difference of two squares is a topic we learnt in school. 20-19 is one, I need 20+19, 20 plus 19, 19 plus 17, … I will add all the numbers from 20 to 1 in this way. (5) A: Yes. (6) Ali: When we consider the two square difference, as 20 minus 19 is equal to 1, 20 plus 19 is equal to 39. There is no importance of 1. (7) A:Ok (8) Ali: I add up 1 at the end and 20 at the beginning. There are 20 of this number sequence but as there are 10 pairs, 21 times 10 is equal to 210. (9) A: What does 210 stand for? (10) Ali: 210 stands for the two square difference here. (11) A: How can you solve it in other ways? (12) Ali: I will draw a square; I will solve it using the area as it is a square. (13) A: Let's see. What are you doing now? Are you drawing a square with 20 long? (14) Ali: Now, here the square of 20 is equal to its total area, however, the square of 19 is equal to the area I have drawn, thus, only this external area remains to me. (15) A: How have you found 39? (16) Ali: 20 is available where I have drawn the line. As 1 of them is common, I omitted it. 20 +20=40. I subtracted 1 and 39 remained.	Semantic Knowledge In the 10th, 14th, 22nd and 24th lines. Strategic Knowledge In the 4th, 12nd, 18th and 26th lines.

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	(17) A: Yes, then? (18) Ali: I will subtract the square of 17 from the square of 18. When I subtract the square of 17 from the square of 18, here will remain. I will again subtract the square of 15 from the square of 16 and 31 remains for me. This decreases by fours and we have 10 in total. (19) A: Ok, what will you do now? (20) Ali: I will sum up these data. I have found 182. I had found 210 before. There should have been a mistake in addition. (21) A: Did you find 4 lower beginning from 39? (22) Ali: We can use that number sequence here. I have found the same result. (23) A: What does 210 stand for? (24) Ali: The solution of the operation. (25) A: How can you show us that you did it correctly or not? (26) Ali: I can review the operations once more in order to make sure about the correctness. Yes, 210.	Schema Knowledge In the 2nd and 4th lines Algorithmic Knowledge In the 4th, 6th, 8nd, 16th, 18th and 22th lines. He could not use in the 20th line.
Veli	(1) A: What are you planning to do before solving the problem? (2) Veli: We can solve it using exponential numbers. (3) A: Yes. (4) Veli: We can take the square root as well. (5) A: What are you planning to do now? (6) Veli: I will try to take the square root. (7) A: Let's go on. (8) Veli: We can think of large number sequences using smaller numbers. (9) A: Let's think about it. (10) Veli: Minus 3 square plus 2 square equals to minus 1 square; minus 9 plus 4 minus 1; here minus 10 plus 4 equals to minus 6. When I take 4 numbers, I will be able to get a different solution with the same type. Then, I can think according to this. (11) A: Do you mean that you will start with simple numbers? (12) Veli: That is, 5 plus 3 equals to 8 now. 16 minus 9 equals to 7, not 5. I always make the this kind of simple mistakes. (13) A: Let's think a bit more. (14) Veli: What have we done, 3, 7, 11. What kind of numbers are these, I will call them as Fibonacci sequence, but it is not. Because, it should have been 10. That is, I thought it might be a similar sequence. (15) A: What are the numbers? (16) Veli: 7, 11, 15. (17) A: Ok. (18) Veli: 15, 19, 23. Then, 27. (19) A: Sum up, then. (20) Veli: Let's go on. 31, 35 … It makes 210 when summed up. (21) A: Can we try another way? (22) Veli: I can solve using the difference of two squares.	Semantic Knowledge In the 40nd line. Strategic Knowledge In the 8th, 22nd, 26th and 38th lines. He could not use in 2nd, 4th and 6th lines.

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	(23) A: Ok. (24) Veli: 2 minus 1 and multiply with 2 plus 1. (25) A: Let's go on. (26) Veli: 4 minus 3, 4 plus 3. One of the numbers is absolutely 1? (27) A: Ok. (28) Veli: The other one will be the sum of these, 39. (29) A: Let's write then. (30) Veli: 39, 35. That is, I caught the 4 difference again. (31) A: Ok. (32) Veli: 35, 31. We have caught the difference here, do I have to do the others? (33) A: You don't. (34) Veli: If I subtract 4, I get 27, 23, 19, 15, 11, 7, 3. (35) A: What's the sum? (36) Veli: I will add up them once more. I might have done it incorrectly. It is 233 but I had found 210. It looks like as it is incorrect. (37) A: Add up the numbers once more, then. (38) Veli: As I sequenced the numbers correctly, there is a problem with the addition. Here is 10, and here is 26. The result is 210, again. (39) A: What does 210 stand for? (40) Veli: The results of the operation I completed.	Algorithmic Knowledge In the 10th, 12nd, 14th, 16th, 18th, 20th, 24th, 26th, 28th, 30th, 32nd, 34th, 36th, and 38th lines. Schema Knowledge In the 14th, 30th, and 32nd lines.
Ayşe	(1) A: What did you understand from the problem? (2) Ayşe: The numbers of which the squares are taken and operations related to them. (3) A: How will you solve the problem? (4) Ayşe: I could understand the difference of two squares now. (5) A: Let's start solving the problem. (6) Ayşe: Here is 30. (7) A: What does it make there? (8) Ayşe: 5. (9) A: Let's go on. (10) Ayşe: Here comes 31. Then, this decreases by fours (11) A: Let's go. What will you do? (12) Ayşe: I will decrease by fours; and then I will sum up. (13) A: Ok, let's sum up. (14) Ayşe: It makes 210. (15) A: What did you do? Did you use the difference of two squares? (16) Ayşe: It also looks like an arithmetic sequence.	Semantic Knowledge In the 2nd line. Strategic Knowledge In the 12nd line. Schema Knowledge In the 4th, 10th, and 16th lines.