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# How come educational inequality in Cuba is much higher than in Chile?

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# How come educational inequality in Cuba is much higher than in Chile?

# ¿Cómo es que la inequidad educacional es mucho más alta en Cuba que en Chile?

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Abstract: "Educational inequality", defined as between-school deviation of academic performance, is much higher in Cuba than in Chile according to a recent UNESCO study. This fact is highly surprising, since income inequality in Cuba is much lower than in Chile. Here we present an agent-based computational model that explains this empirical fact. The model incorporates two key factors: family and school influences, and school selection. When schools are selected by academic performance, educational inequality generated is higher than when schools are selected by socioeconomic status. By comparing urban and rural deviations we find evidence of school selection in Cuba.

**Keywords:** educational inequality, school selection, agent-based models

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### 1. Introduction

According to Beenstock (2012) Rousseau was probably the first to place economic inequality on the philosophical agenda. His contemporary Adam Smith, appalled by the huge economic inequalities, foresaw education as the only mechanism to improve unskilled workers' conditions. Nowadays, educational inequality is a key educational concern all over the world (Green et al., 2003; OECD, 2011; OECD, 2010; Darling-Hammond, 2010; Perry, 2008). It is therefore critical to measure educational inequality and understand how it is generated. Although Schlicht & Ackermann (2010) cannot identify a single concept of educational inequality, they do offer a measure based on the fact that an individual's academic performance is dependent on economic, social, and cultural variables. However, economic or social variables are sometimes difficult to measure uniformly across different countries, and educational inequalities can even emerge when no apparent economic inequalities are present. Therefore, it is very important to have measures of educational inequality that are expressed exclusively in terms of educational variables. For example, Green et al. (2003) use the test score ratio of the mean score in prose literacy for those who have completed tertiary education, compared to those who have not completed upper secondary education. Another example of a measure computed exclusively in terms of educational variables is the standard deviation between schools in regard to academic performance through standardized tests (OECD, 2010). These measures account for educational inequalities even when no apparent economic or social inequalities exist, or can be easily measured. In this way educational inequality can be compared across countries, with completely different economic and social structures.

There have been two UNESCO studies (UNESCO, 1998; UNESCO, 2010a) comparing the performance of several Latin American elementary school students on the subjects of language, math and science. On both studies Cuban performance has been much higher than the rest of the countries. To our knowledge the only analysis of educational inequality has been the computation of the correlation between a socioeconomic and cultural index (ISEC), and performances on the math, language, and sciences tests, both for students and school averages (UNESCO, 2008). In that study, in Cuba the correlation is zero. In all the other countries the gradient of the linear regression differs greatly from zero when computed for school averages. However, when the linear regression is computed for individual students and not school averages, the gradients are much smaller and the share of the performance variance that can be attributed to ISEC is also much lower than when computed for school averages.

### 2. Data and methods

### 2.1 What about standard deviations between schools?

In this work we use the recent Serce UNESCO (UNESCO, 2010a) comparative study of 12 Latin American nations to compare educational inequality across these countries. In each one a random sample of more than 150 schools was selected by UNESCO. In these schools, all third and sixth graders took a math and language assessment test, previously designed by UNESCO experts. The students' tests and student, teacher, and parent surveys were undertaken during 2006. Performance in Cuba was much higher than in all other countries. Cuba was followed by a cluster of countries, including Chile, Costa Rica, Uruguay, and Mexico. Then came a cluster of lower performing countries, including Argentina, Colombia, and Brazil. Finally, Ecuador, El Salvador, Guatemala, Nicaragua, Panama, Paraguay, Peru, and the Dominican Republic formed the final cluster of very low performing countries.

This report confirms a previous UNESCO study (UNESCO, 1998), and other comparative studies (Carnoy, 2007) which show the high academic performance within Cuban schools. However, the Serce UNESCO study also shows that the difference of the performance between schools in Cuba is much higher than in all the other countries studied. For example, as shown in Table 1, in third grade math tests, the standard deviation between schools in Cuba is 93.48, which is the highest, and far higher than in Chile where the standard deviation is 47.62. In third grade language tests the standard deviation between schools in Cuba is 78.07; also far greater than in Chile where the standard deviation is 43.30. Similarly, in sixth grade math the standard deviation between schools in Cuba is 111.19, and in Chile it is 49.47. In language the figures are 76.01 in Cuba and 46.81 in Chile. These measures of educational inequality seem completely opposite to the correlation analysis computed by UNESCO (UNESCO, 2010).

Country	Students'	Students'	Students'	Schools'	Schools' SD	Schools' SD/	(Schools' SD/	Number	Students	Schools' SD	Schools' SD/
	Mean	SD	SD/	Mean	(between	Schools' Mean	Schools' Mean)/	of schools	tested per	for random	Schools' SD
			Students'		schools)		(Students' SD		school	assignment	random
			Mean				/Student Mean)				assignment *
Argentina	506.21	100.07	0.20	505.09	73.17	0.145	0.73	151	41.54	24.30	3.01
Brazil	515.14	104.85	0.20	496.85	67.69	0.136	0.67	136	40.69	25.55	2.65
Colombia	501.22	82.60	0.16	495.04	52.50	0.106	0.64	183	31.61	22.20	2.37
Costa Rica	544.48	80.17	0.15	533.38	44.58	0.084	0.57	144	32.74	22.76	1.96
Cuba	648.00	131.90	0.20	650.66	93.48	0.144	0.71	186	28.45	36.49	2.56
Chile	527.68	93.21	0.18	516.12	47.62	0.092	0.52	151	38.94	24.16	1.97
Ecuador	470.64	87.23	0.19	468.70	50.13	0.107	0.58	178	29.28	25.98	1.93
El Salvador	494.03	78.07	0.16	478.62	40.36	0.084	0.53	175	42.28	18.46	2.19
Nuevo León	562.76	93.51	0.17	555.33	46.30	0.083	0.50	157	34.46	22.81	2.03
Guatemala	465.65	81.02	0.17	464.26	53.43	0.115	0.66	213	32.31	21.37	2.50
Mexico	543.14	99.49	0.18	528.70	56.52	0.107	0.58	149	31.01	26.13	2.16
Nicaragua	471.74	70.73	0.15	475.30	38.56	0.081	0.54	175	36.48	18.83	2.05
Panama	470.72	78.59	0.17	476.20	55.87	0.117	0.70	145	42.73	22.08	2.53
Paraguay	490.17	103.49	0.21	496.43	72.13	0.145	0.69	194	26.34	27.64	2.61
Peru	487.42	86.27	0.18	466.91	56.97	0.122	0.69	154	30.93	24.33	2.34
Dominican R	400.25	86.08	0.22	401.82	47.31	0.118	0.55	145	30.17	21.33	2.22
Uruguay	528.29	102.77	0.19	522.15	53.50	0.102	0.53	207	33.03	27.66	1.93

Table 1: Third grade math means and standard deviations (SD) of performance on the UNESCO Serce Study

# (\*) We call this ratio the Educational Inequality Index. See Appendix.

One possible explanation for the high standard deviation between schools in Cuba could be the higher academic performance of Cuban students. However, countries with higher performance in general obtained lower variations between schools than countries with lower performance. For example, the Chilean performance was higher than several other countries but the standard deviation between schools was generally slightly lower. On other international comparative studies, like PISA (OECD, 2010) and PIRLS (Willms, 2006), countries with higher performance don't have higher variations between schools. In general, the opposite is true. Darling-Hammond (2011), for example, highlights the case of Finland in which very high scores in PISA are accompanied by very low between-schools variance.

Even if standard deviations are normalized by the size of the mean performance (known as the coefficient of variation CV) as shown in Table 1, Cuban standard deviation between schools of third graders in math is 14.4% of the schools' mean, but in Chile the standard deviation is just 9.2% of the schools' mean. This difference contrasts with the deviation of student performance in the respective countries. In Cuba the standard deviation of student performance is 20% of its mean, and the standard deviation of student performance in Chile is 18% of its mean. Therefore, the difference lies within the schools, not the students. In Cuba the normalized between-schools standard deviation by its mean is 71% of the normalized student deviation by its mean, whereas in Chile it is just 52%. The language test for third graders showed similar results, with 67% in Cuba and 47% in Chile.

Another possible explanation for the much higher standard deviation across schools in Cuba is the country's smaller class sizes. However, the UNESCO study averaged the performance in each school, and school sizes across countries are similar. For example, as shown in Table 2 in urban schools the number of third graders tested in math in the study in Cuba is 44.8, and in Chile 50.8: however, the respective standard deviation between schools in Cuba is 98.4, a standard deviation that more than doubles the one in Chile, where it is 45.8. Similarly, as shown in Table 3 in rural schools the average number of tested third graders on math at each school was 11.42 in Cuba and 11.07 in Chile, but the respective standard deviation between schools were 88.25 and 48.70.

Country	Students' Mean	Student s' SD	Students' SD/ Students' Mean	Schools'	Schools' SD (betwe en schools)	Schools' SD/ Schools' Mean	(Schools' SD/ Schools' Mean)/ (Students' SD /Student Mean)	Number of schools	Student s tested per school	Schools SD for random assignme nt	Schools' SD/ Schools' SD random assignment *
Argentina	510.38	97.40	0.19	514.03	55.12	0.11	0.56	101	54.76	16.42	3.36
Brazil	526.36	104.71	0.20	526.60	66.28	0.13	0.63	76	61.14	19.00	3.49
Colombia	507.37	80.31	0.16	507.00	41.65	0.08	0.52	98	45.54	15.96	2.61
Costa Rica	550.87	78.90	0.14	548.34	40.01	0.07	0.51	72	50.50	18.04	2.22
Cuba	649.83	135.23	0.21	645.13	98.37	0.15	0.73	95	44.76	24.76	3.97
Chile	530.21	93.25	0.18	522.82	45.76	0.09	0.50	106	50.77	17.92	2.55
Ecuador	473.32	89.27	0.19	476.48	44.75	0.09	0.50	97	40.98	19.84	2.26
El Salvador	506.48	78.39	0.15	501.77	37.25	0.07	0.48	79	64.29	13.52	2.76
Nuevo León	569.57	93.18	0.16	565.82	40.81	0.07	0.44	97	43.44	18.07	2.26
Guatemala	478.46	79.35	0.17	488.63	43.97	0.09	0.54	85	49.04	17.09	2.57
Mexico	554.89	99.62	0.18	546.39	53.74	0.10	0.55	84	42.93	20.00	2.69
Nicaragua	471.36	70.56	0.15	470.55	27.01	0.06	0.38	89	57.08	12.86	2.10
Panama	475.44	77.44	0.16	486.83	44.47	0.09	0.56	62	78.82	14.43	3.08
Paraguay	494.64	99.78	0.20	497.45	60.07	0.12	0.60	80	42.60	19.61	3.06
Peru	498.62	85.52	0.17	492.64	52.20	0.11	0.62	93	42.06	20.08	2.60
Dominican R.	404.57	87.38	0.22	407.65	41.32	0.10	0.47	78	38.96	17.05	2.42
Uruguay	529.36	103.07	0.19	526.32	53.06	0.10	0.52	156	41.52	20.58	2.58

Table 2: Third grade math means and standard deviations (SD) of performance on the UNESCO Serce Study for urban schools.

(\*) We call this ratio the Educational Inequality Index. See Appendix.

Country	Students'	Students	Students' SD/	Schools'	Schools'	Schools' SD/	(Schools' SD/	Number	Students	Schools'	Schools'
	Mean	' SD	Students'	Mean	SD	Schools'	Schools' Mean)/	of	tested	SD for	SD/ Schools'
			Mean		(betwee	Mean	(Students' SD	schools	per	random	SD random
					n		/Student Mean)		school	assignmen	assignment
					schools)					t	*
Argentina	475.08	113.39	0.24	487.04	98.45	0.20	0.85	50	14.82	39.10	2.52
Brazil	456.33	83.91	0.18	459.16	48.11	0.10	0.57	60	14.78	25.42	1.89
Colombia	480.46	86.74	0.18	481.25	60.07	0.12	0.69	85	15.54	28.65	2.10
Costa Rica	522.91	80.69	0.15	518.41	44.13	0.09	0.55	72	14.97	26.49	1.67
Cuba	640.51	117.06	0.18	656.44	88.25	0.13	0.74	91	11.42	40.64	2.17
Chile	500.42	88.43	0.18	500.33	48.70	0.10	0.55	45	11.07	32.61	1.49
Ecuador	462.04	79.78	0.17	459.38	54.72	0.12	0.69	81	15.27	29.27	1.87
El Salvador	466.77	70.02	0.15	459.58	32.13	0.07	0.47	96	24.17	19.42	1.65
Nuevo León	538.80	90.70	0.17	538.36	49.84	0.09	0.55	60	19.95	27.76	1.80
Guatemala	445.97	79.61	0.18	448.08	53.16	0.12	0.66	128	21.20	23.13	2.30
Mexico	501.40	87.07	0.17	505.84	51.97	0.10	0.59	65	15.62	28.20	1.84
Nicaragua	473.22	71.37	0.15	480.21	47.33	0.10	0.65	86	15.16	23.57	2.01
Panama	453.09	80.38	0.18	468.27	62.15	0.13	0.75	83	15.77	26.80	2.32
Paraguay	481.23	110.03	0.23	495.71	79.76	0.16	0.70	114	14.93	33.68	2.37
Peru	435.92	69.35	0.16	427.70	38.87	0.09	0.57	61	13.95	23.54	1.65
Dominican R.	390.41	82.24	0.21	395.04	52.95	0.13	0.64	67	19.93	24.35	2.18
Uruguay	509.08	95.36	0.19	509.37	53.33	0.10	0.56	51	7.06	39.24	1.36

Table 3: Third grade math means and standard deviations (SD) of performance on the UNESCO Serce Study for rural schools.

# (\*) We call this ratio the Educational Inequality Index. See Appendix.

Another possible explanation for the higher educational inequality in Cuba than in Chile is the fact that in Cuba one teacher teaches all subjects and follows a class from first grade through sixth grade. This means that the students end up with the same teacher for a total of six years. However, math classes are highly uniform as they are partially delivered by TV to the whole of Cuba simultaneously, several times a week. After watching the TV transmission, the teacher helps the students with the corresponding textbook's exercises. Furthermore, a strict teacher evaluation program

is performed in the country, and teachers whose students do not perform well on the local standardized test are sent to training programs in local universities for one year. If, after this training, their students do not perform well, the teacher has to leave the profession. All of these facts suggest that teacher quality is highly uniform in Cuba, possibly much more uniform than in Chile. In addition, the standard deviation between schools in the urban sector is 3.97 times that of school deviations when students have been randomly assigned (see Appendix). This is much higher than in the rural sector, where the corresponding figure is only 2.17. Therefore, if the high standard deviation between schools is due to the impact of teachers, it remains to be explained why this is only true for urban schools. Normally, the opposite would be expected, since schools and classes are much smaller in the rural sector, and the average performance of the school on a grade level corresponds to a single teacher. This is what happened in the Serce UNESCO study, shown in Table 3, where the average number of students in each grade in Cuban rural schools was 11.42. Therefore, this school average most likely corresponds to the influence of a single teacher, whereas in the urban sector the number of students was 44.76, and thus the school average most likely corresponds to the influence of at least two teachers.

In this work we explore another possible explanation of the higher educational inequality in Cuba as opposed to Chile. We explore the possibility that school selection mechanisms could be implicitly or explicitly being used in the two countries, as well as the influence of family and school on academic performance (Brunner & Elacqua, 2005; Carnoy, 2007; Willms, 2008). The challenge is to explain how a very high between-school standard deviation on the tests can be generated, while a very low socioeconomic deviation is present at the same time.

### 2.2 Agent-based model with selection mechanism based on academic performance

We are interested in finding a very simple model that can show how inequality arises. As in Heinrich, J. & Boyd (2008), we expect that the fewer key factors there are, the better the understanding of inequality and segregation will be. We seek to explain how inequality emerges in very short time scales: just two or three generations. Therefore, asymptotic behavior and/or an equilibrium state reached in the long term are irrelevant (Epstein, 2006). We propose an agent-based mathematical model and run computer simulations as in Maroulis et al., (2010b). For reasons of simplicity we only model intergenerational behavior, instead of a much finer grained model that could take into account the dynamics from one year to the next (Maroulis et al., 2010a). We will see that this simple agent- based model is enough to generate the pattern of educational inequalities that we are trying to understand.

The model has two key mechanisms: one for intergenerational academic performance where family and school are critical, and another for school selection by families. The simulations explain the possibility of higher educational inequality in societies with no socioeconomic inequality than the ones with high socioeconomic inequality. The basic idea is that segregation arises as a result of individual choices (Schelling, 1969). In this particular case, we assume the existence of school selection mechanisms as part of a

natural, nepotistic behavior in nonhuman and human animals (Maestripieri, 2012; Bellow, 2003), that has been present in human beings since prehistory (Bentley et al., 2012). Nepotism, or favoritism towards kin, is a universal phenomenon. In the model, nepotism means school selection by the family even when it could not be officially permitted or recognized. We will see that school selection, after only two or three generations, along with very simple intergenerational education dynamics, generates a clear pattern of school segregation.

For reasons of simplicity we will assume that all families have just one member in each generation, and that families are mono-parental. Thus, let;

p(i,n) be the academic performance of the member of family i of generation n

e(i,n) be the school the generation n member of the family i attends

 $\bar{P}(e,n)$  be the average academic performance of all students attending school e in generation n

The model assumes that the academic performance of the member of the new generation is a combination between his parent's academic performance (genetic and family influence) and the school and peer influence (but augmented by an effectiveness factor), plus a random term that accounts for other influences.

$$p(i,n+1) = \alpha \ p(i,n) + (1-\alpha) \ \overline{P}(e(i,n+1),n+1) \ \pi + \varepsilon \xi(i,n)$$
$$p(i,0) \ is \ N(\mu,\sigma)$$

where  $\xi(i,n)$  are independent N(0,1),  $\varepsilon$  is the standard deviation of the unknown influences,  $\alpha$  is a number between 0 and 1 representing the relative weight of family influence against school and peers, and  $\pi$  is a number representing the academic effectiveness of the school (a number close to 1. Here we used  $\pi$  = 1.05). This equation is similar to ones used for intergeneration income (Gintis & Bowles, 2001).

In each generation the family selects a school. We will compare two selection mechanisms. The first one is based exclusively on academic performance.

$$e(i,n+1)=e(i,n)$$
 if  $p(i,n)\leq \overline{P}(e(i,n),n)+\beta\sigma$  ; 
$$e(i,n+1)=e$$
 a randomly selected school such as  $\overline{P}(e(i,n),n)\leq \overline{P}(e,n)$  in which there is a vacancy

In other words, if the academic performance of the father was higher than the average performance of the school which he attended by  $\beta$  times the initial standard deviation of student performance, then he changes his offspring's school to one with a higher average than the school he attended. He initiates the change only if there is such a school in which there is a vacancy. We also define a maximum size for any school.

### 2.3 Agent-based model with selection mechanism based on socioeconomic status

A second selection mechanism is the one where families implicitly or explicitly select schools based on socioeconomic status. This is the case of Chile (Carnoy, 2007; Mizala et al., 2012; Elacqua et al., 2006). Most families in urban sectors have several schools to choose from in their neighborhood. Parents in low socioeconomic neighborhoods have the option of sending their children to a government subsidized private school, indicating a higher socioeconomic status. Even within state schools, socioeconomic status is important in the selection process. In a recent survey of 480 parents from 16 low socioeconomic and highly vulnerable government-subsidized and public elementary schools (grade 1 through 8) in Santiago, Chile, one of the most common reasons provided for considering changing school was the socioeconomic status of students attending the school.

Thus, let;

s(i,n) be the socioeconomic status of the member of the family i in the generation n

 $\bar{S}(e,n)$  be the average of the socioeconomic status of students attending school e in the generation n

We assume a very simple model for socioeconomic mobility:

$$s(i,n+1) = s(i,n) + \omega \psi(i,n)$$

s(i,0) is  $N(\mu, \sigma)$  (with no loss of generality we assume the same media and standard deviation as in p(i,0) for simplicity of comparing effects)

where  $\psi$  (i,n) are independent random variables N(0,1).

We also assume that there is a correlation between socioeconomic status and academic performance, due to an initial correlation in the generation 0. That is, correlation between s(i,0) and p(i,0) is  $\rho$ . The correlations in the other generations are obtained from the dynamic equation for social mobility and the dynamic equation for academic performance.

The school selection mechanism based on socioeconomic status proposed is also very simple, and with a similar structure to the one based on academic performance

$$e(i,n+1) = e(i,n)$$
 if  $s(i,n) \le \overline{S}(e(i,n),n) + \beta \sigma$ 

e(i,n+1) = e a randomly selected school such as  $\bar{S}(e(i,n),n) \leq \bar{S}(e,n)$  if there is vacancy within such a school.

In other words, if the socioeconomic status of the father was higher than the socioeconomic status of the school he attended by  $\beta$  times the initial standard deviation of socio economic status, then he changes his offspring's school for one with a higher average than the school he attended. He initiates the change only if there is such a school in which there is a vacancy. As before, we also define a maximum size for any school.

### 3. Results

We run the agent-based computer model for several generations but we show here the results for the third generation. The simulation started with a first generation of 6,000 students randomly assigned to 60 schools. The maximum number of students that each school can have was set to 1,000. 31 different values of  $\beta$  in [-1.5, 1.5] and 31 different values of  $\alpha$  in [0, 1] were used. For  $\mu$  we used 250 and for  $\sigma$  we used 50, both are the mean and standard deviation on Chilean state assessments for fourth graders. For  $\varepsilon$  we used 10, for  $\pi$  we used 1.05, and for  $\omega$  we used 15.

**Finding 1**: After a couple of generations, simulations of the agent-based computer model show that educational inequality grows in a society with a school selection mechanism based on academic performance.

Thus, educational segregation results from individual choices of educational type. This school selection mechanism is probably implicitly used in societies like Cuba, where schools are free and there is almost no income inequality. It is conjectured that families probably move to another neighborhood in order to be assigned to a good school (what is called residential school choice (Loeb et Al, 2011)), or seek some other strategy to select schools.

**Finding 2:** After a couple of generations, simulations of the agent-based computer model show that the dispersion of academic performance between schools where the selection mechanism is based on academic performance is higher than the dispersion of academic performance between schools where the selection mechanism is based on socioeconomic status.

We compute the ratio between the standard deviation of academic performance between schools when selection is based on academic performance over when it is based on socioeconomic status. Figure 1 shows this ratio in the third generation for  $\rho$  = 0. The ratio is higher for  $\beta$  and closer to zero (change to a school simply because it has a better academic average) and for  $\alpha$  closer to zero (where schools have more influence than family).

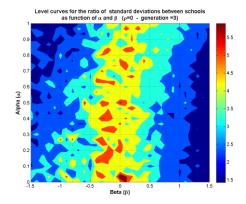


Figure 1: Ratio of standard deviations between schools as function of  $\alpha$  and  $\theta$ , for  $\rho$ =0

Now we explore the effect of the correlation between academic performance and socioeconomic status.

**Finding 3:** The bigger the initial correlation  $\rho$  between academic performance and socioeconomic status, the smaller the ratio between the standard deviations.

This means that when the correlation between academic performance and socioeconomic status is high, then both selection mechanisms, by academic performance and by socioeconomic status, have similar impacts on the educational inequality. As shown in Figure 2, there is a relation between  $\alpha$ ,  $\beta$ , and  $\rho$ . It may be possible to estimate one of them from the other, as well as the ratio of the standard deviations.

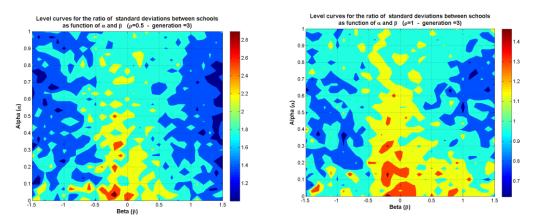


Figure 2: Ratio of standard deviations of academic performance between schools as function of  $\alpha$  and  $\beta$ , for  $\rho=0.5$  and 1. Note that the color scales are different for different  $\rho$ , and therefore different from Figure 1.

# 4. Discussion

### 4.1 Evidence of school selection in Chile

As mentioned before, there is ample evidence of school selection based on socioeconomic status in Chile, and therefore schools have high homogeneity in the socioeconomic status of their student body (Carnoy, 2007; Mizala et al., 2012; Elacqua et al., 2006). In a recent study, where we surveyed 480 parents from 16 low socioeconomic status schools, parents selected the socioeconomic status of the students attending the school as an important factor in moving their children from one school to another. For example, in Lo Prado, an urban district of low socioeconomic status, we interviewed 30 parents from each of the 11 public elementary schools in the district as well as 30 parents from each of the two private subsidized schools in the district. 40% of the parents from the private schools knew other parents from their neighborhood that wanted to bring their children to the school and 33% mentioned socioeconomic status as one of the three main reasons for doing so. In the public schools, 24% of the interviewed parents knew other parents that wanted to bring their children to the school, and 18% mentioned socioeconomic status as one of the three main motivating reasons. On the other hand, 26% of the parents of the 11 public schools of the district recognized that other schools from their district had advantages over the school that their children attend. Socioeconomic status was mentioned 24% of the time, second only to performance on state tests, which was mentioned 31% of the time.

## 4.2 Evidence of school selection in Cuba

What evidence do we have of possible school selection by families in Cuba? One type of evidence is obtained from comparing urban and rural schools. It is expected that in rural schools there are far fewer options available in terms of possible schools to which to send third and sixth graders. Normally there is just one elementary school nearby. On the other hand, in the urban sector families have multiple options. Therefore, if in a country a school selection mechanism is present then it must operate in the urban sector with higher frequency and intensity than in the rural sector. Thus, it is very informative in each country to compare the variation of performance across schools in the urban sector with the respective variation in the rural sector.

The Serce UNESCO study classified schools as belonging to either an urban or a rural sector, and therefore we can use this classification to compare sectors. However, one important problem is that rural schools are much smaller than urban ones. Particularly in the UNESCO study, the average number of students tested in each rural school is generally much smaller than the average number of students tested in urban schools. For example, in Cuba the average number of students tested on math in third grade is 11.42 students per school; in Chile it is 11.07 students per school. In the urban sector the average number of students tested per school is 44.76 in Cuba and 50.77 in Chile. Thus, it is expected that this big difference naturally generates a much higher standard deviation across rural schools than in urban ones. It is therefore necessary to consider this effect and to look for a way in which to compensate the difference between urban and rural sectors in terms of the number of students tested per school.

We propose comparing the variation of performance across schools with the ideal case of students being assigned to schools completely at random. Using the standard deviation of the performance of the student tested in a sector, the number of students tested in each school and the number of schools tested within the sector, the exact standard deviation across schools in the ideal case of random assignment can be computed.

Thus, let  $\Omega$  be the standard deviation of the performance on a test of the students tested in a sector (urban or rural), let m be the number of school tested, and let  $n_i$  be the number of students tested in the school i, then the standard deviation of the performance across schools in the ideal case of completely random assignment is (see Appendix):

$$\frac{\Omega}{\sqrt{m}}\sqrt{1-\frac{1}{m}}\sqrt{\sum_{i=1}^{m}\frac{1}{n_{i}}}$$

We then compute the ratio between the standard deviation between schools of the performance on a test in a given sector, over the corresponding standard deviation in the ideal case of completely random assignment of students to schools. We call this ratio the *educational inequality index* of a sector.

As shown in Table 2, in third grade math tests in the urban sector the educational inequality index in Cuba is 3.97, whereas in the rural sector it is 2.17 (see Table 3). This means that there is a much higher educational inequality index in urban schools than in rural schools. In the urban sector in Chile the educational inequality index is 2.55, and in the rural sector it is 1.49. As such, in Chile there is also a difference in the educational inequality index between urban and rural schools, albeit a little more subtle than in Cuba.

Note that the educational inequality index captures this big difference between urban and rural sectors. By simply comparing deviation between schools, deviation between schools normalized by their means (coefficient of variation), or the proportion of deviation between schools as part of the student deviations such difference is not detected. For example, in Cuba the deviation between schools in the urban sector is 98.37, while the figure for the rural sector is 88.25. The between-school coefficient of variation in the urban sector is 0.15, whereas in the rural sector it is 0.13. In the urban sector the coefficient of variation between schools is 73% of the coefficient of variation of the students, while in the rural sector it is 74%. All three of these classic measures of inequality would erroneously suggest that there is no significant difference between urban and rural sectors. But these measures do not take into account the fact that schools are much smaller in the rural sector, and that the number of students tested in each rural school is much lower than in urban schools. This fact is very concerning given that school size has a major impact on deviations across schools, and therefore must be taken into consideration. The educational inequality index is precisely the corrective tool needed to take this impact into account.

One could interpret the big difference in the educational inequality index between urban and rural sectors in Cuba as the influence of schools in Cuba is very important, and that it is higher than in Chile. However, this would imply that there are some schools with a much higher aggregate value than others, and only in the urban sector. It would have to be explained why this happens only in the urban sector. We have already argued that it cannot be the effect of teachers. On the other hand, Serce UNESCO data shows a similar educational inequality index in the third and sixth grades, for math tests both in urban and rural sectors. For example, in the urban sector in third grade math the educational inequality index is 3.97, and 3.95 in the sixth grade. In language the pattern is similar. In the urban sector the educational inequality index is 3.49 in the third grade and 3.22 in the sixth grade. We do not have the data to estimate the proportion of students that move from one school to another, but by analyzing data from interviews that we have conducted with local authorities, in Cuba students move to other schools mainly after sixth grade. Therefore, we can assume that in each school the type of student is similar in third and sixth grade. Thus, the fact that educational inequality indices for third and sixth

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grades are very similar means that the differential impact of school does not increase during these 3 years. This is also true also in Chile. Even though the students are not the same, if the difference between schools is due to the aggregate value of some urban schools, it would be expected that the difference would increase the longer the students stay on in the schools. In the language test the trend is similar, although in all countries the educational inequality index in the rural sector is slightly higher in sixth grade than in third grade.

4.3 Further evidence of school selection in Cuba from deviations between parents' schools

The Serce UNESCO study (UNESCO, 2010) shows that in Cuba there is no correlation between average school performance and average student socioeconomic and cultural status (ISEC). The socioeconomic and cultural index ISEC (UNESCO, 2010b) is a mix of several variables asked about through a survey. In one of the survey questions, third and sixth grade parents reported their level of education. They were required to identify one of seven levels of education per parent. In the following analysis, we will use the education level of the students' mothers since several other studies identify this as being one of the most influential variables for student performance (De Fraja et Al, 2005) and a student's future income (Bjorklund et Al, 2010). Analysis of the mothers' education is very important in order to test the school selection hypothesis, since contrary to student performance, a mother's education is not influenced by teachers or schools. Therefore a high deviation in the education level of the students' mothers across schools is a clear indication of school selection, and particularly so when the deviation is much higher in the urban sector than in the rural sector.

In urban schools in Cuba third grade mothers reported, on average, one level above Chile (5.73 in Cuba against 4.69 in Chile), whereas in rural schools the difference was one and a half levels (5.06 against 3.55). This measure has some drawbacks. Levels are described in terms of elementary and secondary education, but these terms convey different meanings in terms of years of study across different countries. The analysis of the deviation across schools of the mothers' education supports the hypothesis that schools are selected in Cuba, and that the selection would be done by families according to their education. The educational inequality index computed for the mothers' education is much higher in urban schools than in rural schools. For example, the standard deviation of the mothers' education across schools, normalized by the standard deviation in the ideal case of random school assignment, shows that for sixth graders taking the math test in Cuba in the urban sector, the ratio for their mothers is 2.20 (Table 4). This is much higher than in the rural sector, where the respective figure is just 1.00 (Table 5). This means that the mothers' education is random across schools in the rural sector, and therefore the distribution across schools is the same as the natural distribution of the mothers' education in the rural sector. However, this is not true in the urban sector. The distribution of the mothers' education across urban schools shows a statistically significant difference from the natural distribution of the mothers' education in the urban sector. Some schools have a concentration of mothers with more education and other schools have concentrations of mothers with less education. The inequality index of the mothers' education is 2.20 times higher in urban schools than in rural schools. In the urban sector in Chile this inequality index of the mothers' education is 3.17, and in the rural sector 1.28, and therefore the ratio between the inequality indexes of the mothers' education is 2.48. This is not too different from the 2.20 ratio obtained in Cuba.

Country	Mothers'	Mother	Mothers' SD/	Schools'	Schools'	Schools' SD/	(Schools' SD/	Number	Mother	Schools'	Schools'
	Mean	s' SD	Mothers'	Mean	SD	Schools'	Schools' Mean)/	of	s′	SD for	SD/
			Mean		(betwe	Mean	(Mothers' SD	schools	respons	random	Schools' SD
					en		/Student Mean)		es per	assignme	random
					schools)				school	nt	assignment
											*
Argentina	4.50	1.67	0.37	4.42	1.00	0.23	0.61	98	42.20	0.35	2.87
Brazil	3.43	1.90	0.55	3.50	1.38	0.39	0.71	62	67.69	0.35	3.99
Colombia	3.64	1.93	0.53	3.74	0.96	0.26	0.48	56	73.57	0.32	3.00
Costa Rica	4.19	1.63	0.39	4.23	1.12	0.27	0.68	68	37.87	0.46	2.46
Cuba	5.68	1.18	0.21	5.51	0.49	0.09	0.43	98	46.21	0.22	2.20
Chile	4.46	1.59	0.36	4.41	1.00	0.23	0.64	109	44.97	0.32	3.17
Ecuador	4.18	1.63	0.39	4.04	0.87	0.22	0.55	94	32.09	0.43	2.04
El Salvador	2.28	1.94	0.85	2.21	0.98	0.44	0.52	82	45.34	0.47	2.08
Nuevo Léon	4.73	1.55	0.33	4.58	0.87	0.19	0.58	96	36.86	0.33	2.64
Guatemala	2.84	1.66	0.58	2.94	1.02	0.35	0.59	85	34.89	0.39	2.62
Mexico	-	-	-	-	-	-	-	0	-	-	-
Nicaragua	3.58	1.81	0.50	3.68	0.84	0.23	0.45	93	43.74	0.41	2.06
Panama	5.08	1.58	0.31	5.25	0.89	0.17	0.55	57	63.95	0.32	2.78
Paraguay	4.28	1.79	0.42	4.15	1.05	0.25	0.60	75	31.31	0.44	2.40
Peru	4.47	1.73	0.39	4.45	1.09	0.25	0.64	87	41.40	0.43	2.54
Dominican R	4.37	1.95	0.45	4.44	0.99	0.22	0.50	61	32.87	0.40	2.46
Uruguay	4.09	1.43	0.35	3.93	0.88	0.22	0.64	156	33.19	0.32	2.78

Table 4: Educational level of mothers whose sixth grade children took the math test in the UNESCO Study for urban schools.

(\*) We call this ratio the Educational Inequality Index for education level of the students' mothers. See Appendix.

Country	Mothers' Mean	Mother s' SD	Mothers' SD/ Mothers' Mean	Schools' Mean	Schools' SD (between schools)	Schools' SD/ Schools' Mean	(Schools' SD/ Schools' Mean)/ (Mothers' SD /Student Mean)	Number of schools	Mother s' respons es per school	Schools' SD for random assignme nt	Schools' SD/ Schools' SD random assignment
		1.51		2.24	0.00	0.20	0.52		0.20	0.70	*
Argentina	3.18	1.51	0.48	3.01	0.89	0.29	0.62	56	9.39	0.72	1.23
Brazil Colombia	3.09	2.03	0.56	2.28	1.06	0.18	0.32 0.55	15 32	34.33 34.09	0.32	1.29 2.02
Costa Rica	3.34	1.49	0.45	3.12	0.75	0.36	0.54	64	11.20	0.52	1.24
Cuba	4.81	1.49	0.45	4.75	0.47	0.10	0.38	95	10.45	0.61	1.00
Chile	3.45	1.56	0.45	3.50	0.83	0.10	0.52	48	11.52	0.47	1.28
Ecuador	3.09	1.42	0.46	2.82	0.75	0.24	0.58	71	12.85	0.62	1.21
El Salvador	1.44	1.32	0.40	1.25	0.73	0.27	0.38	85	13.54	0.60	0.73
Nuevo León	3.97	1.52	0.38	3.75	0.77	0.33	0.53	62	15.60	0.56	1.38
Guatemala	1.92	1.22	0.63	1.75	0.57	0.20	0.51	107	13.15	0.41	1.39
Mexico	-	-	-	-	-	-	-	0	-	-	-
Nicaragua	2.69	1.66	0.62	2.51	0.82	0.33	0.53	61	13.34	0.64	1.28
Panama	3.64	1.71	0.47	3.24	1.02	0.32	0.67	87	11.31	0.71	1.43
Paraguay	2.98	1.56	0.52	2.84	0.85	0.30	0.57	104	10.25	0.61	1.39
Peru	2.81	1.39	0.50	2.58	0.71	0.27	0.55	53	12.62	0.51	1.38
Dominican R	3.22	1.70	0.53	3.09	0.74	0.24	0.46	34	20.59	0.49	1.51
Uruguay	3.36	1.07	0.32	3.35	0.48	0.14	0.44	55	5.55	0.50	0.96

Table 5: Educational level of mothers whose sixth grade children took the math test in the UNESCO Serce Study for rural schools.

(\*) We call this ratio the Educational Inequality Index for education level of the students' mothers. See Appendix.

How then is it possible that in Cuba there is no correlation between the socioeconomic cultural (ISEC) index and school performance? One possible explanation is that correlations are not accurate enough when there is too little variation in one variable. In this case, the dispersion of ISEC in Cuba is very small. It ranges from -0.7 to 1, which is much smaller than in Chile where it goes from -0.7 to 2. If we look for an interval of the same range of variation in Chile where most schools score, for example, ISEC from 0 to 1.5, then in that segment the correlation is also close to zero. Thus, the educational inequality index proposed in this paper (see Appendix) captures possible inequalities more accurately than the correlation. It is more accurate since it does not depend on other variables external to education. These external variables are sometimes difficult to measure with accuracy, or they vary too little in one country but not in another.

Another problematic point with the simple linear regression computed for school averages of performance and socioeconomic index, is that it fails to take into account the enormous variance of school performance as well as the variation in the average education of the parents at that school. In particular, it does not account for the difference in the variation of student performance and the parents' education between schools, which is caused by the wide variation across schools of the number of students tested in each school. If a new sample of similar schools of identical size were to be taken, then the schools' student performance and parent education averages may change dramatically for rural schools. The educational inequality index, on the other hand, does take into account this difference in the number of students per school, across different urban and rural schools. One way to account for the variation in school sizes in the regression is to compute the linear regression of the school average as function of the average mother's education but weighted by the number of students with both data in each school. If computed in this way, the correlation for Cuba's urban sector is not zero; in fact, the gradient is 24.88, with R<sup>2</sup> = 0.011, and is statistically significant. This means that for a single-level increase in the education level of the students' mothers there is a corresponding increase of 24.88 points in the school's average student performance on a third grade math test.

### 5. Conclusions

International comparative studies are important natural experiments, particularly across countries in which the same language is spoken and a similar culture is shared. They can help us to explore and analyze in order to understand possible key mechanisms behind educational phenomena. In this paper we proposed an explanation of an interesting and surprising fact: that educational inequality in Cuba is greater than in Chile and the rest of Latin America. The explanation is obtained by running an agent-based model that uses two key factors: influence of family and school on academic performance; and the mechanism of school selection. According to Beenstock (2012), "surprisingly, economics has largely ignored the role of family as a source of inequality". In the model presented here, the natural nepotistic behavior of families is expressed in two different ways according to the options available in the

respective societies. In Cuba it is expressed in school selection by families based on academic performance, and in Chile in school selection by families based on socioeconomic status. The conjecture of the existence of this particular mechanism of school selection in Cuba is supported by the big difference in educational inequality indices in Cuba between the urban and rural sectors, both for the students' performance and for the education level of their mothers. By generating a computer-based model of two types of societies with the same intergenerational mechanism of academic performance, in which family and school play the critical roles, but with different school selection mechanisms, we obtain a plausible explanation of the far greater educational inequality present in Cuba as compared to Chile.

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## Appendix: educational inequality index

In order to estimate the degree of educational inequality we propose comparing the student data gathered with the data that would be gathered in the ideal case in which students are assigned completely at random to the existing schools included in the sample. More explicitly, we have a fixed sample of schools, each one with a known number of students to be tested, and we have to assign to these schools students selected from the sector's entire population. For each school, students are selected at random and assigned to the school in question. We assume that the number of students in the sector is large compared to the total number of students to be assigned. Thus, we define the educational inequality index as the ratio between the standard deviation between schools with the standard deviation between schools obtained in the ideal case of completely random assignment. Moreover, since it is expected that in the rural sector there are fewer schools from which to select, we propose comparing the educational inequality index in the urban sector with that of the rural sector.

Let  $p_{ij}$  be the performance of student j from school i, then;

 $P_i=rac{1}{n_i}\sum_{j=1}^{n_i}p_{ij}$  is the performance of the school i obtained from the performance of the students of a sample of  $n_i$  students from this school.

Let us assume that the students' performance  $p_{ij}$  are independent random variables with mean  $\eta$  and standard deviation  $\Omega$ . These are the mean and standard deviation of the students of the sample belonging to a whole sector, or the entire country.

Therefore, the expected value of each school is also  $\eta$ ;

$$E(P_i) = \frac{1}{n_i} \sum_{j=1}^{n_i} E(p_{ij}) = \eta$$

And the variance of school i is;

$$V(P_i) = \mathsf{E}(P_i - \eta)^2 = E(\frac{1}{n_i} \sum_{j=1}^{n_i} p_{ij} - \eta)^2 = E(\frac{1}{n_i} \sum_{j=1}^{n_i} p_{ij} - \frac{1}{n_i} \sum_{j=1}^{n_i} \eta)^2$$

and because of independence, the variance is =  $\frac{1}{n_i^2} \sum_{j=1}^{n_i} E(p_{ij} - \eta)^2 = \frac{\Omega^2}{n_i}$ 

Therefore, the standard deviation of the performance at school i is  $\Omega(P_i) = \frac{\Omega}{\sqrt{n_i}}$ 

Let  $P=\frac{1}{m}\sum_{i=1}^{m}P_{i}$  be the mean performance of a sample of m schools of a given sector.

Then, the variance of the performance between schools is;

$$\frac{1}{m}\sum_{i=1}^{m}(P_i-P)^2$$

Let us compute its expected value.

$$= \frac{1}{m} E(\sum_{i=1}^{m} (P_i - P)^2) = \frac{1}{m} \sum_{i=1}^{m} E(P_i - P)^2 = \frac{1}{m} \sum_{i=1}^{m} E(P_i - \eta + \eta - P)^2$$

$$= \frac{1}{m} \sum_{i=1}^{m} [E(P_i - \eta)^2 + 2 E(P_i - \eta)(\eta - P) + E(\eta - P)^2]$$

Using the independence of the school performance (that comes from the independence of the performance of the students and the random assignment of students to schools) we have;

$$= \frac{1}{m} \sum_{i=1}^{m} \left( \frac{\Omega^2}{n_i} - \frac{2\Omega^2}{mn_i} + \frac{\Omega^2}{mn_i} \right) = \frac{\Omega^2}{m} \left( 1 - \frac{1}{m} \right) \sum_{i=1}^{m} \frac{1}{n_i}$$

Therefore, the expected value of standard deviation of the performance between schools is;

$$\Omega(P_i, i=1, \dots, m \ ) = \frac{\Omega}{\sqrt{m}} \sqrt{1 - \frac{1}{m}} \sqrt{\sum_{i=1}^m \frac{1}{n_i}}$$

Then on a sector where the student population has a performance with a standard deviation  $\Omega$  on a test taken on m schools to  $n_i$  students of the school i, the educational inequality index of the sector is defined as;

The standard deviation between schools of student performances  $/\frac{\varOmega}{\sqrt{m}}\sqrt{1-\frac{1}{m}}\sqrt{\sum_{i=1}^{m}\frac{1}{n_i}}$ 

The educational inequality index is scale-independent. If all students increase their performance by a given percentage, the education inequality index does not change. In case of complete educational equality obtained by random assignment the educational inequality index is 1. However, the educational inequality index can be less than one. For example, it is less than one when students are cherry-picked to be assigned to schools in order to obtain schools which are very similar in educational performance.

The educational inequality index can be computed for other variables besides test performance. We have computed it for the parents' education. In this case we call it the educational inequality index for parents' education.